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Abstract

In twentieth century the quantum theory of physics has been a fascinating playground to study the nature of electromagnetic radiations and matter. In this subject, the forces on atom by light have received much theoretical and experimental attention during past many years, not only because of interest in the basic atom field interaction, but also for the measurement of an unknown state of electromagnetic field which poses an interesting question in it. The measurement of the cavity field had gained a very high attention because of the possibility of the quantum computers, quantum teleportation, quantum cryptography, dense coding and many more.

A project of Pakistan Science Foundation entitled "Quantum State Measurement" is taken to keep our research in this area. There are many schemes presented for the quantum state measurement. One of the most widely used ways is the reconstruction of Wigner function. We worked in this area and presented new schemes for the reconstruction of Wigner function of the field from the recovered photon statistics of the field. Photon statistics can be recovered in no of ways. In this report we present five different new schemes for the measurement of photon statistics of the field. These are based on Deflection of atomic beam from the cavity field in Raman-Nath regime, Electromagnatically induced transparency, Resonance florescence, Ramsey interferometry, Autler-Towns time dependent spectroscopy, and Deflection of atomic beam in Bragg's regime.

In the atomic beam deflection in Raman-Nath regime the momentum distribution of the atoms after their interaction during the passage through the quantized cavity field is used for its reconstruction. We displace the photon statistics of the cavity field and reconstruct the Wigner function of the Schrodinger-cat state. In the Electromagnatically induced transparency we use a three level atom, the upper two levels were driven by the quantized field. The absorption spectrum of the probe beam gives the information about the photon statistics while in Resonance Florescence, instead of three level atoms we use two level atoms driven by the field. If the driving field is position dependent then we find the position of the atoms passing through the cavity in Sub wavelength domain. In Ramsey interferometry we proposed to measure the joint photon statistics in two cavities containing entangled field. The cavities are placed in between the two Ramsey fields and two level atoms pass through these cavities and two Ramsey zone. In this setup the atoms goes under a dispersive phase shift while their passage through the off resonant entangled cavities. By measuring the internal states of the atoms we can reconstruct the photon statistics and then the Wigner function. The Autler-Towns spectroscopy is the reverse of EIT where the upper two levels of a three level atoms are driven by the field. Instead of measuring absorption spectrum we measure the spontaneous emission spectrum. In another scheme of atomic beam deflection in Bragg regime we measure the momentum distribution of atoms after passing through the two cavities containing entangled field. The momentum states contain the information about the joint photon statistics.

Apart from these schemes we also proposed another schemes for the reconstruction of Wigner function using tomography by phase sensitive amplification of the field. Three level atoms of two photon processes are passed through the cavity amplifying the field to be measured. The two cases are discussed here. One in which the

phase of the atoms are controlled outside the cavity and the other in which the phase is controlled inside the cavity. The complete quadrature distribution is obtained by measuring the quadrature for the different phases. The inverse Radon transformation is then employed to reconstruct the original quantum state.

Most of these schemes are based on the atom field interaction and the role of phase and intensity of the field. In one of our study we consider spontaneous emission in a four-level atomic system driven by three fields. It is shown that, by controlling the phase and the amplitude of the driving fields, a wide variety of spectral behavior can be obtained ranging from a very narrow single spectral line to six spectral lines of varying widths.

We also present an exciting application of new emerging field of Quantum Informatics i.e., Quantum Teleportation. We consider the teleportation of entangled two-particle and multiparticle states and present a scheme for the teleportation that may be suitable for both entangled atomic states or field states inside high Q cavities.

Paper # 1

Measurement of Wigner function via atomic beam deflection in Raman-Nath regime

Introduction

The concept of a quantum state has always played a key role in discussions treating the foundations of quantum theory. Each physical quantity can be represented by a Hermitian operator, which is called an `observable'. A measurement of this observable leaves the system in an eigenstate of the operator. A single measurement performed on a quantum system reveals a certain aspect of its state, and it will not uncover the quantum state completely. However, if we know how to determine the whole set of potentialities, the quantum state can be recovered. It is a basic assumption of the quantum theory that an infinite ensemble of system contains all the information about the quantum system. The quantum state of the radiation field is described completely by the state vector for a pure state and by the density operator for a more general mixed state. Equivalent descriptions of the quantum state can be formulated in terms of the quasiprobability distributions such as P-representation, Q-representation or the Wigner distribution function. These distributions, which do not have all the properties of a classical probability distribution, allow the evaluation of various correlation functions of the field operators, using the methods of classical statistical mechanics. For example, the Wigner distribution function affords the evaluation of symmetrically ordered correlation functions of the creation and destruction operators of the field. In recent years, a large class of the states of the radiation field has been studied. Some of them such as a squeezed state or a Schrodinger-cat state exhibit interesting features in their quantum statistical properties for example they may have oscillatory photon distributions. Several methods have been proposed to measure quantum state of light as well as quantum state of matter. One method that allows us to perform measurements on wave function is the so-called tomographic method. In this method, the distribution for the electric field quadrature amplitudes are measured via optical homodyne measurements and the Wigner distribution function of the radiation field is constructed from these measurements via Radon transformation. This method has been realized experimentally. The other methods, which allow us to determine the quantum state of an electromagnetic field in a cavity or quantum state of matter, are based on the fundamental interaction of atoms with the cavity field. These include methods based on dispersive atom-field coupling in a Ramsey method of separated oscillatory fields, atomic beam deflection and the conditional measurements on the atoms in a micromaser setup. A class of schemes for the measurement of the quantum state of the radiation field involves the measurement of the absorption and emission spectrum in a driven system. The atom deflection method uses momentum distribution of atoms in order to reveal the quantum state of the light inside the cavity. In this case, the atom serves as a tool that probes a quantum state of radiation field.

Freyberger and Herkommer proposed a scheme to measure the quantized cavity field. Their proposed scheme utilizes resonant two-level atoms. The atoms are prepared in coherent superposition of atomic states, before interacting with the cavity field, which

is to be measured. They put a narrow slit of width much smaller than the wavelength of the cavity field in front of standing wave (of the cavity field). Under this approximation, sin(kx) dependence can be replaced by linear kx dependence in the interaction Hamiltonian. They have studied the momentum distribution of the atoms and found the photon statistics of the cavity field by the recursion relation w_m=a/(w_m-1)*. The scheme works well for the case where the probability amplitudes of photon statistics never go to zero. On the other hand it has limitations that w_m cannot be found when the probability amplitude (w_m-1)* becomes equal to zero. Such is, for example, in the case of Schrodinger-cat state, which has an oscillatory photon statistics. Another scheme proposed by Schneider also takes into account the atomic deflection method for the reconstruction of the quantized cavity field. They make use of a strong coherent reference field traveling orthogonally with respect to the cavity mode. The strong coherent field plays the similar role as the local oscillator in the homodyne tomography. While their passage through the cavity, the atomic probe interacts for a short time with the cavity and the reference field. They measure the momentum distribution of the atoms for different phases of the classical field and reconstruct an s-parameterized quasiprobability distribution of the field.

Contrary to Freyberger and Herkommer, the method we propose do not require the superposition of atomic states, rather than this we inject a coherent state inside the cavity which causes the displacement of the original photon statistics of the field. Our proposed scheme has advantage of using mixed state instead of pure state. Furthermore, the scheme probes a beam of two-level atoms in their ground state for the measurement of Wigner function of the cavity field. We observe that the momentum distribution of the atoms, after interacting with the cavity field, contains enough information for the reconstruction of photon statistics and hence the Wigner function of the cavity field. The cavity field is however coupled to a resonant classical oscillator for the measurement of Wigner function. The injection of coherent state causes a displacement of the initial state of the cavity field in phase space. The role of this coupling in the quantum state measurement has been discussed in. The Wigner function is reconstructed in a completely different way and the mathematical framework is more simpler. We derive expression for the Wigner function of the cavity field in terms of displaced photon statistics. We also discuss the role of the derived expression used to reconstruct the photon statistics of the cavity field. Both the resonant and off-resonant atom-field interaction is discussed. We recover the photon statistics of the Schrodinger-cat state and also reconstruct the Wigner function for the same. It is very interesting to note that in non-demolition interaction of the off-resonant case, the mathematics is much simpler.

Wigner function of the radiation field

In order to measure the Wigner function of the cavity field we assume that there is a probability p(m) of m photons inside the cavity. A classical oscillator is connected to the cavity so that it injects a coherent state inside it. The injection of the coherent state causes the displacement of the field. We recall the definition of the quasiprobability distribution, which belongs to a general class of phase-space distribution in the form of s parameters. By the order of operators we mean the normal order, antinormal order and symmetric order. The s=1,-1, or 0 for normal-, antinormal-, and symmetric-order,

respectively. For s=0 one obtains the Wigner distribution function and for s=-1, and 1, Q-representation and P-representation, respectively. The Wigner function of the cavity field can be found directly if the displaced photon statistics is known. In this paper, we first find out the photon statistics and then the Wigner characteristic function of the cavity field with the help of displaced photon statistics.

Resonant atom-field interaction

We consider the same scheme as that of Freyberger and Herkommer with two modifications: (a) we inject two-level atoms in the standing wave cavity field in their ground state and (b) we displace the photon statistics of the cavity field by injecting a coherent state for the measurement of the Wigner function. The transition of the two-level atoms is resonant with the single mode quantized cavity field. A narrow slit placed in front of the cavity allows the atoms to collimate on a small region of the cavity field. Further, we consider the interaction in Raman-Nath regime where the kinetic energy term in Hamiltonian can be neglected. The atoms are transmitted through the opening of the slit. We approximate that the opening is very small as compared to the wave length of the cavity field and is centered around x=0. Due to this approximation we replace the usual sin(kx) dependence of the standing wave of cavity field by a linear kx dependence. We obtain the set of coupled differential equations for density matrix elements and solve using the method of first finding the eigen values and then using them for the solution of coupled differential equations. We have supposed that the photon statistics of the cavity field is p(m), and the atoms are injected in their ground state. This in turn makes the initial condition that the initial probabilities of all the density matrix elements other than the ground state are zero.

The equations of motion of the density matrix elements are solved subject to initial conditions. In order to measure the momentum distribution of the atoms, for the proposed experiment, we take the Fourier transform from position space to momentum space in normalized co-ordinates. The atomic states, the field states and the position states of the atoms are entangled at this stage. We are interested in the momentum distribution of the atoms and hence do not bother about the internal states of the atoms and the cavity field states. For the measurement of the momentum of atomic probe, we take the trace over the field states and the internal states of the atoms, which lead to the probability of the momentum distribution on the detection screen. As an example, while measuring photon statistics of the cavity field we take the normalized Gaussian distribution of the atoms at the slit. We take the example of the Schrodinger-cat state. We recover the photon statistics, which shows good agreement with the original one. We note that when the value of ""kappa"" is increased, the peaks in the momentum distribution spectrum resolve. We recover the photon statistics of the cavity field with the help of resolved spectrum of the momentum distribution. For the reconstruction of Wigner function of the cavity field, we inject a coherent state inside the cavity, which displaces the original photon statistics of the field in phase space. This displaced photon statistics of the cavity field is recovered from the momentum distribution using the same method as discussed above. We reconstruct the Wigner function of the cavity field after finding the displaced photon statistics. The photon distribution of the Schrodinger-cat state after the injection of coherent state alpha is reconstructed. We take the contour plot of the

original Wigner function and reconstruct it. As we begin to increase the value of "kappa", the Wigner function becomes closer and closer to the original one. This behavior of the recovery of Wigner function is just like the same as the recovery of photon statistics.

Off-resonant atom field interaction

In this section we discuss the situation where the standing wave field, inside the cavity, encodes information in the center of mass momenta of the off-resonant out going atoms in Raman-Nath regime. Again we suppose a quantized field in a cavity with a slit in front of it. A beam of two-level atoms having detuning "Delta" interacts with the cavity field. For the Raman-Nath regime, the transverse motion of the atoms during interaction is ignored which allows us to drop the kinetic energy term in the Hamiltonian. The issue involved in this section is essentially used for analyzing quantum non-demolition measurement. For the sake of simplicity, we use the effective Hamiltonian in our calculations. In this case, the off-diagonal density matrix elements are absent. Furthermore, the equations of motion for density matrix elements of the excited and ground state atoms are independent of each other, thus we can formally integrate them to get the result. The supposition, that the atoms are in ground state and they are highly detuned with the cavity field leads us to conclude that they remain in ground state after the interaction with the cavity field. Following the same procedure as in the case of resonant atom-field interaction, we take the Fourier transform of the equation from position space to momentum space in normalized co-ordinates. While we propose to focus the atomic beam in the mid way between node and anti-node of the standing wave cavity field. We use the small opening of the slit as compared to the wavelength of the standing wave cavity field and choose the same initial condition as described in the resonant atom-field interaction. We get the momentum distribution of the atomic probe by taking the trace over the field and atomic states.

In the present case, the peaks of the momentum distribution become resolved at very small value of "kappa" as compared to the resonant case. We reconstruct the photon statistics of the Schrodinger-cat state. For the case of small "kappa" the peaks of the momentum distribution are not fully resolved which causes the partial recovery of the photon statistics. The situation is more favorable if we consider relatively higher value for "kappa", where the photon statistics is fully recovered. For the recovery of the Wigner function, we displace the photon statistics of the cavity field by injecting the coherent state from local classical oscillator. The displaced photon statistics is recovered from the momentum distribution, and is then used to reconstruct the Wigner distribution function of the cavity field.

Results and Discussions

We have recovered the photon statistics of the cavity field in case of Schrodinger-cat state for both the resonant and off-resonant case. The graphs are taken for initial Gaussian distribution of the atoms at the slit. First, while considering the resonant atom-field interaction, we note that when "kappa" is increased from 25 to 45 gradually, the peaks in the momentum distribution spectrum begin to resolve. The opening of the slit has the reverse behavior on the momentum distribution spectrum, i.e., as we decrease the

width of the slit, the more strength of the coupling is needed. We have used the limit that the slit has smaller width as compared to the wavelength of the cavity field. This allows us to replace the sin(kx) dependence by linear kx dependence. We use slit width 1/4 of the wavelength of the cavity field, which is a valid approximation under the existing limit. One can find the photon statistics of the cavity field with the help of the resolved spectrum of the momentum distribution. Adopting the same procedure we recover the photon statistics of the cavity field. For kappa=25 the peaks of the momentum distribution are not fully resolved which causes the partial recovery of the photon statistics of the cavity field. Although the positions of the peaks in recovered photon statistics match with the original but the heights remain different. The photon statistics is fully recovered when kappa=45 where the peaks of the momentum distribution are fully resolved. For the case of off-resonant atom-field interaction, we need smaller values of "kappa" as compared to the resonant case and are enough to resolve the peaks in momentum distribution. The values used for the slit opening and the photon statistics of the cavity field are same as they were used in the case of resonant atom-field interaction, but here we need just kappa=10 for the best reconstruction of the photon statistics as discussed earlier in the previous section.

In order to recover the Wigner function, we displace the photon statistics of the cavity field by injecting the coherent state by a local classical oscillator. We recover the Wigner function of the cavity field for resonant atom field interaction. It has recovered completely for kappa=45. However, for lower values of "kappa", the partial recovery of the Wigner function is observed. We also reconstruct the Wigner function of the cavity field for the off-resonant atom-field interaction. In the off-resonant case for "kappa"=10, we get the full recovery of the Wigner function.

In conclusion we have proposed a scheme for the reconstruction of the photon statistics and hence for the Wigner function of the quantized cavity field for both the resonant and off-resonant atom-field interaction. We observe that in case of off-resonant atom field interaction a small value of "kappa" is sufficient to observe the resolved peak spectrum in momentum distribution of out going atoms. While on the other hand, we need relatively large value of "kappa" for the case of resonant atom-field interaction. The advantage of the presented scheme is that it works well for the case of mixed state also. The parameters chosen for the reconstruction of the photon statistics and the Wigner function are attainable in current state of art.

Paper # 2

Quantum state tomography using phase-sensitive amplification

Introduction

Quantum state measurement has been a subject of great interest in recent years. As all the knowable information of a quantum system are contained in the density matrix of the system, so the measurement of the density matrix elements will completely characterize the given quantum state. The Wigner function of a quantum state bears a one to one correspondence with the density matrix of the state. Once the Wigner function of a quantum state is known then the corresponding density matrix elements of the state can

be worked out by employing the Wigner formula. On the measurement side, a balanced homodyne detector measures the linear combination of the creation and the annihilation operators of a quantized field. This linear combination of creation and annihilation operators is also termed as the generalized or rotated quadrature of the field and the phase of this quadrature is given by the phase of the local oscillator in the balanced homodyne detection scheme. Vogel and Risken have shown that the quasiprobability distributions such as P, Q, and Wigner function bear a one to one correspondence with the generalized quadrature distribution function. From a set of measurements of the generalized quadrature amplitude in the balanced homodyne detection scheme, the quadrature distribution can be known and hence by tomographic imaging of this distribution, the P, Q, and the Wigner function can be obtained. Following the same scheme, Faridani and later Mlynek have experimentally measured the quantum state of the radiation field. Recently, some other methods have also been proposed for the measurement of quantum state of the radiation field. These include methods based on absorption and emission spectroscopy, the conditional measurements on the atoms in a micromaser, dispersive atom-field coupling in Ramsey method of separated oscillatory fields beside some others. In some recent studies, it is shown that the measured quadrature distribution becomes smoothed due to the finite detection efficiency. As a result, instead of Wigner function smoothed quasiprobabilities are constructed.

In this paper, we propose a scheme for the measurement of quantum state of the radiation field using two-photon CEL. During the amplification through a phase-sensitive amplifier, there is no noise in the quadrature of interest and all the noise is fed into the conjugate quadrature. Therefore, the quantum information remains intact in one quadrature phase of the field and may be extracted out of it for the construction of quantum state of the field. In order to construct the Wigner function of the quantum state, we require a set of distribution functions for quadrature values. To obtain noise free amplification for different quadrature phases, we prepare the amplifier in different phases, accordingly. We have calculated the quadrature distributions for any arbitrary quantum state after its amplification through a phase sensitive amplifier. The distribution function of the noise free quadrature is then used to construct the Wigner function of the quantum state using quantum tomography. We apply this model to a Schrodinger-cat state and discuss the reconstruction of the corresponding Wigner function after its amplification through a two-photon CEL. Our proposed method is insensitive to detector efficiency which poses serious problems in observing the non-classical features associated with the quantum state. In a recent paper, we have shown that the quantum interferences associated with a Schrodinger-cat state can be observed using phase-sensitive linear amplification. It may be pointed out that the phase-sensitive amplification of the Schrodinger-cat state and the resulting non-classical characteristics during the amplification process are discussed in the references.

Measurement of the Quantum state using two-photon CEL

We consider a two-photon phase sensitive linear amplifier, which consists of three-level atoms in cascade configuration. The atoms are initially prepared in a coherent superposition of upper most and lower most levels. We assume that such atoms are injected at a random injection rate inside the cavity where they interact with the field. It is

assumed that the cavity field is resonant with the atomic transitions. We write the master equation for the reduced density matrix as the following Fokker-Planck equation for the Wigner function. A solution of this equation yields the evolution of the Wigner function for any arbitrary initial quantum state. Here we look at the measurement of the quadrature distribution for the amplified quantum state. A complete distribution can be given by the quadrature distribution. Such distributions have recently been measured employing quantum optical tomography. The quadrature distribution for the amplified field can be obtained from the Wigner function. The solution indicates a one to one correspondence between the phase of the atomic coherence and the phase of the field quadrature. In order to reconstruct the Wigner function of the initial quantum state, we need a set of distribution function for different values of theta varying from 0 to pi.

The Wigner function can be constructed by amplifying the signal such that there is no noise in the desired quadrature and all the noise is fed into the conjugate quadrature. To obtain the noise free amplification, we prepare the atoms in a coherent superposition with a particular phase. The atoms are then injected inside the cavity where they amplify the initial quantum state. The noise free quadrature can be obtained by adjusting the phase of the local oscillator. To find the complete set of distributions, we prepare the amplifier for a set of values of atomic coherent superposition phases ranging from 0 to 2pi and obtain noise free amplification for the desired quadratures.

Once the quadrature distributions of the amplified signal are measured in balanced homodyne measurement, then the complete Wigner function is determined by carrying out the inverse Radon transformation familiar in tomographic imaging. For sufficiently large squeezing, we obtain the same original state for any arbitrary value of the gain parameter. This shows that the proposed scheme allows us to fully reconstruct the original quantum state after its amplification through a phase-sensitive linear amplifier. However, an appropriate rescaling of the measured distribution is required. As an example, we consider the Schrodinger-cat state. It is clear that for sufficiently large squeezing, and for any arbitrary value of the gain parameter, we obtain the Wigner function for the initial Schrodinger-cat state, which is quite interesting.

Results and discussion

The Wigner function shows two Gaussian hills at the location of two coherent states and oscillations on the conjugate axis due to the superposition of two coherent states. This is the well known behavior associated with the Schrodinger-cat state. It is shown that the well known oscillations due to the Schrodinger-cat state vanish when it is amplified through a phase insensitive amplifier. However, for r=1 and 2 the oscillations start appearing which is quite interesting. For strong enough squeezing, we almost fully recover the Wigner function corresponding to initial Schrodinger-cat state. These results confirm our assertion that amplifying the signal with the help of a phase-sensitive linear amplifier allows us to fully reconstruct the original quantum state. The Wigner function is reconstructed by taking the inverse Radon transform, once the quadrature distributions are measured after amplification through two-photon CEL. The quadrature distributions can be measured using balanced homodyne detection scheme. The parameters in the experiment should be adjusted such that field leakage through the end mirror does not occur during the amplification process.

In conclusion, we propose a scheme to measure the quantum state of the radiation field. The technique is based on amplifying the signal with the help of a two-photon CEL such that there is no noise in the quadrature of interest. Our scheme is insensitive to problems associated with the detector inefficiencies. In a recent paper Lenohardt and Paul have also proposed an interesting scheme based on antisqueezing the field with respect to the desired quadrature using degenerate optical parameter amplifier that also allows to overcome the problem of detector efficiency.

Paper # 3

Quantum Teleportation of an Entangled State

Introduction

The notions of coherent superposition and entanglement in quantum mechanics lie at the conceptual foundation of quantum mechanics as evident through fundamentals contributions like Bell's inequalities and Greenberger-Horne-Zeilinger (GHZ) equalities. These novel concepts are finding interesting and useful applications in the field of quantum computing and quantum information. One of the key problems in quantum communication is the transmission of a quantum state from one observer to another and to keep the received state exactly the same as that sent while no information carrier needs to move. This can be accomplished in two steps. First, the sender "disassembles" information of a quantum state into two parts, one of which is sent through a quantum channel run by the non-local correlation between two entangled quantum entities and the other is sent through a classical channel. Second, the receiver "reconstructs" the state on the basis of the information from both the quantum and classical channels. Because in this process a quantum state to be transmitted is destroyed in one place and later it is reconstructed in another place, this transmission is termed as teleportation of a quantum state. Bennett proposed a scheme for the teleportation of an unknown quantum state from one observer to another through duel Einstein-Podolsky-Rosen (EPR) and classical channels. Since this proposal, a number of experimentally feasible schemes have been suggested for the teleportation of two-level atomic states and field states for two-dimensional states to N-dimensional states. Most of these schemes rely on methods based on cavity quantum electrodynamics in which two identical high-Q cavities are initially prepared in an entangled state. Quantum teleportation was experimentally verified by producing pairs of entangled photons by the process of parametric down conversion. Recently a scheme has been presented which exploits the cavity decay for teleportation of atomic state of an atom trapped in a leaky cavity. Beside these schemes of discrete variables much progress has also been made for quantum teleportation of states of dynamical variables with continuous spectra. The teleportation of a coherent state of the radiation field and teleportation of superposition of amplitudes have also been reported. All these schemes are for the teleportation of single qubit states. In many potential applications of quantum computing, such as factorizing a very large number or searching an unordered quantum database, one needs the system of many qubits states. It is therefore an interesting question whether we can teleport a multi qubit state. In this paper,

we present a scheme for the teleportation of two-particle entangled state from a pair of high Q cavities to another pair of high Q cavities using methods based on cavity quantum electrodynamics. This scheme is then generalized for the teleportation of N-qubits field state.

Quantum teleportation of entangled state

We consider the teleportation of a two qubits entangled state of the radiation field in two separated high Q cavities to another pair of high Q cavities. It may be pointed out that this scheme corresponds to the teleportation of entangled two-level atomic states also as the atomic entanglement can be transferred to the two cavities by passing them through the two cavities with pi pulse. The teleportation of state can be carried out in three steps.

In the first step, we prepare two pairs of cavities in entangled states. Passing a two-level atom initially in the excited state through the two resonant cavities can do this. The interaction times of atom with two cavities are chosen such that we have a pi /2 pulse in the first cavity and a pi pulse in the second cavity. Initially the two cavities are taken in vacuum and two level atoms in excited state. When atom has undergone a pi/2 pulse in first cavity, the second cavity is still empty and the atom-field system is in a state which corresponds to a linear superposition with equal weights of atomic states correlated to zero and one photon, respectively. If atom is still in excited state after leaving first cavity, it will, with unit probability, be transferred to ground and leave a photon in second cavity. If it emits a photons in first cavity and exit it in ground level, it will be unaffected by its coupling with the vacuum in second cavity in ground state. Thus atom always exits from second cavity in ground, while the field is left in the entangled state. Similarly we prepare another pair of cavities in entangled state.

The second step of the teleportation is the measurement of Bell states. There are number of ways for the determination of number of photons inside the cavities. We propose to use Ramsey Interferometry. In this scheme we consider two-level atoms initially prepared in ground state and which are off resonant with the radiation field inside the cavities. The cavities are placed between two classical microwave fields. When atom passes from first zone with a microwave field tuned to atomic frequency, it is prepared in a coherent superposition of states. This atom is then passed through the two selected cavities with the same interaction time in each cavity. During the passage through the cavities, a phase shift proportional to the photon numbers in the two cavities is introduced as a phase of the state b. The atom is then passed through the second zone again resonant. The interaction time and the coupling parameters are chosen such that a→(a+b)/sqrt{2} and b→(a-b)/sqrt{2}. It is however clear that a measurement of the atom would reduce the fields inside the cavities to states with only appropriate number of total photons in the two cavities. The first atom is sent through the two cavities with interaction time pi in each cavity. It follows that if the atom is found in the excited state, the total number of photons in the two cavities is even. If the atom is detected in state b then the total number of photons in the two cavities is odd. In the next step we make measurement in the cavities only with same interaction time. We consider two two-level atoms initially in their ground states. One of the atoms is sent through the cavity B_{1} and the other through cavity B_{2}. After the passage, the atomic internal states a and b are mixed by a classical field. A subsequent detection of these atoms introduces phase factors. By a

similar procedure the photon can be removed from the cavity and the resulting cavity field state will have phase factors according to the final outcome of the atomic state. The resulting state can have different but known phase factors between the constituent states. The net effect is equivalent to a transformation to a different basis. Next we make measurements in the cavities in order to determine the phase factors. We can summarize from above that if the order of detection of first two atom is the same as last two then we have j_{1}=0 and j_{2}=0. If the detection of atomic states are same for first and third atom and detection of fourth atom is reversed w.r.t. second atom then j_{1}=0 and j_{2}=1. If the detection of atomic states are same for second and fourth atom and detection of third atom is reversed w.r.t. first atom then j_{1}=1 and j_{2}=0. If the order of detection of atomic states for third and fourth atom are reversed with respect to the first and second atom respectively, then j_{1}=1 and j_{2}=1.

In the final step of the teleportation, we transform this state into the original state. Transformation involves two steps. One is the removal of phases and the other is an appropriate transformation of photon numbers. First we consider the transformation of phase only. An atom in a superposition state is passed through the cavity C_{2} only in such a way that the ground state picks the phase while the excited state does not pick any additional phase. If the atom is detected in ground state after the passage through cavity C_{2} then the state (A_{1}A_{2}) is recovered, If atom is detected in state a then repeat the process until the atom is detected in b.

In order to recover the original state, we should interchange the state between 0 and 1 photon in cavity C_{2}. For this purpose, we pass a two level atom in its ground state through cavity C_{2} with a pi pulse followed by its passage through a classical field again with a pi pulse (a→ b and b→ a) and finally through an empty cavity C_{2} such that the atom in excited state leaves the cavity in ground state while leaving 1 photon inside the cavity and the atom in ground state leaves the cavity in the ground state with no photon inside the cavity. This leads to the field states in the cavities in entangled state and the teleportation is complete.

Teleportation of n-qubit field state

After giving a scheme to teleport 2-qubit state, we would like to generalize this scheme for N-qubits state. These N entangled pair of cavities can be prepared as mentioned earlier by passing two-level atoms initially in excited state through the two resonant cavities and by setting pi/2 pulse and pi pulse respectively in the two cavities. We now make measurement of the 2^{2N} basis states of the system. It has 2N parameters, N parameters correspond to phase, while the remaining N parameters correspond to the photon numbers inside the cavities. Thus the state can be determined in two sets of measurement, the first determining the set of the relative phases. For the determination of photon numbers we use Ramsey interferometry. Detection of atom either in the excited state or in the ground state makes the probable outcomes of total N values. We then send second atom in ground state with same interaction time, which reduces the probable outcomes to further half. Similarly we continue the procedure. A detection of atom either in the excited state or in the ground state completely determines the values of outcomes.

For the determination of phase factors we follow the same procedure as for 2-qubit state earlier. We send N two-level atoms initially in ground state one by one through the cavities. After the passage through the cavity a classical field mixes the atomic internal states. A subsequent detection of these atoms introduces phase factors yielding possible outcomes of atomic states. Each combination has 2N outcomes of atomic states, N outcomes each for evacuation of B_{n} and A_{n}. If all the outcomes of first N are reversed with all the outcomes of last N of that combination then we have all j equal to 1. This completes the procedure of measuring the Bell states.

First we consider the transformation of phase only. It depends upon the value of j. If all j are 0, then we have to do nothing and original state is recovered. However if any j_{n} among N values of j is 1 then it has additional phase with it. For the removal of this phase we send a two-level atom in a coherent superposition of states through the cavity in such a way that ground state picks the phase. If the atom is detected in b then original state is recovered otherwise we have to repeat the process until it is detected in b. If there are m values of j that are equal to 1 out of N values of j then we pass m atoms in coherent superposition of states one by one from those m cavities and detect the atom in ground state. If all the j are 1 then we pass N atoms in (a + b)/sqrt{2} from all N cavities and detect atoms in ground state.

Next we consider the transformation of photon numbers in the cavities. This transformation depends upon the values of k. If all the k are 0 then we have to do nothing and the original state is recovered. However if any k_{n} among N values of k is 1, then we have to change 0 and 1 photon from cavity C_{n}. For this purpose we pass a two level-atom in its ground state through cavity C_{n} with a pi pulse followed by its passage through classical field again with a pi pulse. Finally the atom passes through an empty cavity C_{n} such that the atom in excited state leaves the cavity in ground state while leaving 1 photon inside the cavity and the atom in ground state leaves the cavity in ground state with no photon inside the cavity. This leads the field states in the cavities in the entangled state and the teleportation is complete. If there are m values of k that are equal to 1 out of N values then we repeat the same process as above by sending m two-level atoms one by one in ground state from each m cavity and proceed further as mentioned earlier till the completion of the process. If all the k are 1 then we pass N atoms in ground state from all N cavities followed by a classical field that mixes a and b as a→b and b→a and finally through N empty cavities. This completes the transformation process and hence the teleportation of N -qubit state.

Conclusions and results

We have presented a scheme for the quantum teleportation of a 2-qubits entangled state from a pair of cavities at the sender's end to another pair of cavities at the receiver's end. The scheme employs atomic interaction with high Q cavities. We need two entangled states of two particle each for the teleportation of two particle entangled state. Sending one particle of each entangled state to sender and other particle to receiver is sufficient to teleport the entangled state of two-qubits. This scheme is then generalized for the teleportation of N-qubits entangled state in N high Q cavities. For this purpose we need N entangled states of 2-qubits each. Sending one particle of each entangled state to sending station and other particle of that state to receiving station is enough for the

teleportation process. The proposed scheme of teleportation consists of three steps. The first step involves preparation of quantum entangled states between two high Q cavities. The second and third steps involve optical Ramsey interferometry and single photon transfer. All these require controlled interaction times between atoms and cavities, negligible cavity loss and no spontaneous decay during the whole teleportation process. Controlling the interaction time in the cavities can easily be achieved by properly setting, through Stark field adjustment, the times during which atom is resonant with each. About the spontaneous decay we propose the Rydberg atom in circular states with principle quantum number approx 50. They have a long radiative life time (30 ms) and a very strong coupling to radiation. A negligible cavity loss is also required during the whole process of teleportation. Cavity life times for high Q cavities should be long enough as all the interactions of atom with cavities should be completed before the cavity dissipation. High quality factors of such cavities and control of atomic beams during the whole teleportation process may pose limitations on the suggested scheme.

Paper # 4

Amplitude and Phase Control of Spontaneous Emission

Introduction

Spontaneous emission in atomic systems arises due to the interaction of atoms with the environmental modes. It is an interesting area of research to consider various means and systems to modify and control the spontaneous emission spectrum. We can control the fluorescence spectra by placing an atom in a frequency dependent reservoir, by placing the atoms in microwave cavities, or near the edge of photonic bands gaps. For atoms in free space, atomic coherence and quantum interference are the basic mechanisms for controlling the spontaneous emission. A control of spontaneous emission in atomic system via quantum interference and atomic coherence results in a number of novel phenomena such as lasing without inversion, electromagnetically induced transparency, correlated spontaneous emission laser, absorption cancellation and enhancement of the index of refraction with no absorption. The quenching of spontaneous emission in an open V-type atom was studied. Phase dependent effects in spontaneous emission spectra in a Lambda-type atom were presented. Recently Paspalakis and Knight proposed a phase control scheme in a four level atom driven by two lasers of the same frequencies, where the relative phase of the two laser was used to get partial cancellation, extreme linewidth narrowing and total cancellation in the spontaneous emission spectrum. In these calculations, parallel dipoles for the two transitions were assumed. However, orthogonal dipoles for the two transitions with small energy separation are easy to be found in nature. Therefore, it is worth to consider the spectral linewidth narrowing and other effects for the case of two orthogonal dipoles, by controlling one relative phase and keeping another relative phase constant.

In this paper we present another scheme for four-level atom in which we can control the spontaneous emission by the amplitude and the phase of the driving fields. In our scheme, the quantum coherence is generated by a microwave field instead of the

sharing of the vacuum modes by the two transitions. The proposed scheme requires three driving fields but is more convenient in its experimental realization. We present analytical results for the spontaneous emission spectrum of a four-level atom. The upper two levels are closely spaced and are driven by microwave field. These two levels are coupled with a third level via two coherent fields and decay to the fourth level. All the interactions are assumed to be resonant. We study the various effects of the dynamical variables namely the amplitudes or, more precisely, the Rabi frequencies and carrier phases of the driven fields on the spontaneous emission spectrum. We predict six peaks spectral behavior, which are sensitive to these variables and their control results in extreme partial cancellation and extreme linewidth narrowing. The linewidth narrowing is seen in the central peaks of the two sets of dressed-states originating from slow decay rates. This is in agreement with the work of Zhou and Swain where they found linewidth narrowing in one of the dressed-states nearby the quenching condition in the context of resonance fluorescence of a closed V-type atom. In this paper we assume that the transition frequency between the upper two levels is large as compared to their decay rates. This approximation allows us to neglect the quantum interference term in the equations of motion for the probability amplitudes. Our system is therefore independent of the alignment of the dipole moments. The trapping conditions is, however, not physically achievable in this approximation.

Model

We consider a system of four-level atom interacting with three driving fields. These fields resonantly couple the transitions a1 - b, a2 - b, and a1 - a2. The upper levels a1 and a2 decay to the lower level c via interactions with the vacuum field modes. We assume the coupling constants to be real for the sake of simplicity. If the matrix elements are orthogonal, there is no interference between the decay paths a1 - c, and a2 - c, and if the matrix elements are parallel, there is maximum interference. To analyze the spontaneous emission spectrum we assume the atom to be initially prepared in the state b.

The spontaneous emission spectrum consists of two parts. Each part corresponds to three peaks associated with three dressed-states, which is composed. We neglected the interference terms between the two sets of dressed-states corresponding to the two bare-states due to large separation between them. The spectrum therefore consists of six peaks. In any situations of interest, the interference terms occurring in the spectrum equation have negligible contributions. We examine the condition for trapping in this system. In order to have a non-vanishing steady-state population in the upper states of the system, the constant term of its characteristic equation is set to zero.

Results and discussion

Our system reduces to the usual form of Autler-Townes scheme where the spontaneous emission spectrum is split in doublet, when the atom is initially prepared in the state a2 and the Rabi frequencies and the decay rates are equal to zero. If the atom is initially prepared in the coherent superposition of the upper two level and the decay rates are non-zero there are four peaks in the spectrum originated from dynamical stark splitting of the upper two levels. The variation of the relative phase of the pump and

driving lasers result in a similar effect as the one reported recently. The spectrum contains two major parts due to the two bare upper states. Each part contains three terms corresponding to three peaks associated with the three dressed-states in each. The equation therefore leads to a spectral behavior consisting of six peaks. The interference terms have negligible contributions. We consider the effects of the dynamical variables on the spontaneous emission spectrum. The variation of the phase associated with the microwave field influences the spontaneous emission spectrum efficiently. In the spectrum all the terms, except the central terms, are. The plot for these values shows an extremely suppressed central peak and enhanced side peaks. Furthermore, for the two bare-states, the height of one side peak is larger than the other. The central terms grow up to their maximum when phi is varied from 0 to pi /2. In this case, the plot shows a suppressed central peak and equally enhanced side peaks for the one bare-state and vice versa for the second. When phi is further varied from pi /2 to pi, the new spectrum is just the mirror inversion. We note, that the peaks height varies with phi, however there is no appreciable change in the position of the spectral lines on the frequency axes. This behavior is in agreement with the coherently driven three-level Lambda-type atom of Martinez. The enhancement around pi/2, 3pi/2 and strong suppression around 0, pi of the central peaks is in accord with the work of Paspalakis and Knight reported recently where they used the relative phase of two lasers of the same frequencies to control the three peaks spontaneous emission spectrum in a four levels atom.

The shape of spontaneous emission spectrum is strongly influenced by the variation of the Rabi frequencies. For instance, when the Rabi frequency Omega3, is reduced to 0.1 and phi =0, the contributions of the central terms are negligible. This is also evident from the plot that the central peaks are suppressed extremely and the side peaks are enhanced, moreover, the height of one side peak is slightly larger than the other. The central terms still remain negligible when phi is varied from 0 to pi /2. We find an extremely suppressed central peak but equally enhanced side peaks. When phi is further varied from pi /2 to pi, the new spectrum is just the mirror inversion and we get the mirror inversion if phi is varied to 3 pi/2. The decrease (increase) of the Rabi frequency Omega3, depopulates (populates) the central dressed-states of the two bare-states and therefore, for the optimum value of the Rabi frequency, the spontaneous decay rates become negligible (maximum).

It is worthwhile noting that the height of the central peaks increase and the side peaks decrease, with the decrease of the Rabi frequencies. In addition, the width of the central peaks gets extremely narrow for the low value of the Rabi frequencies whereas the side peaks are suppressed almost completely. The plot shows extremely suppressed side peaks and equally enhanced central peaks. Moreover, a remarkable spectral narrowing is also seen. By varying phi from 0 to pi, one of the central terms increases while the other decreases. This is also clear from the plot that the central peaks is enhanced with the height of a peak is larger than the other. When is further varied from pi/2 to pi, the new spectrum is just the mirror inversion and we get the mirror inversion if is varied to 3pi/2. We note that width of the lines emitting from the central dressed-states become extremely narrow when the Rabi frequencies are further reduced. This spectral narrowing is associated with the slow decay rates. The behavior agrees with Zhou and Swain in obtaining the linewidth narrowing of one of the dressed-states nearby the quenching condition in the context of resonance fluorescence of a closed V-type atom.

In summary, we have shown that by chosing appropriate parameters for the amplitude and the phase of the driving fields we can obtain a very wide variety of spectral behavior ranging from a very narrow spectral line to upto six spectral lines of varying widths. The present system is very easily experimentally realizable.

Paper # 5

Measurement of Photon Statistics via Electromagnetically Induced Transparency

Introduction

Electromagnetically induced transparency (EIT) was first observed by Harris. Since then this effect has been studied intensively both theoretically and experimentally. The theoretical studies assume the driving field to be classical. In this paper we study EIT by a quantized driving field inside a cavity and show that the absorption spectrum provides a direct means of measuring the photon statistics of the field. This method of measuring the photon statistics has the advantage that the photon statistics of the radiation field can be directly measured from the spectrum without resorting to cumbersome numerical manipulations of the experimental data. In addition, the proposed method is insensitive to the detector efficiency, which poses serious problems in observing non-classical characteristics of the field. The diagonal elements of the density operator with respect to the Fock state give the photon distribution function. The photon distribution for many fields may demonstrate novel non-classical features such as an oscillatory behavior in the case of single-mode squeezed vacuum state or the Schrodinger-cat state. It is a problem of recent interest to experimentally observe such non-classical features of the quantum state of the radiation field. Quantum state of the field is also determined by using optical homodyne tomography, which uses measured distributions of electric field quadrature amplitude to determine the Wigner function and hence the density matrix. From the knowledge of the density matrix, information about photon number and phase distributions is obtained. It has also been realized experimentally. Other schemes include methods based on dispersive atom-field coupling in a Ramsey method of separated oscillatory fields, atomic beam deflection, the conditional measurements on the atoms in a micromaser set-up, the Autler-Townes spectroscopy, resonance fluorescence, homodyning, unbalanced homodyning, photon chopping, and photon counting.

In this paper, we propose a scheme to determine the photon statistics of the radiation field inside a cavity using a set up that is employed in the observation of EIT. In EIT, a three-level atomic system is considered. When a classical field drives the upper levels, the medium becomes transparent for a probe field resonant with the lower level transition. The transparency results from the combined Stark splitting and quantum interference of the dressed states, which are created by applying that additional electromagnetic field. The splitting of the level is proportional to the associated Rabi frequency. Heights of the peaks of the absorption spectrum are independent of the Rabi frequency. Peaks are displaced from resonance by an amount equal to Rabi frequency. If

the upper levels are being driven by a quantized field, the associated Rabi frequencies are distributed according to the photon distribution of the driving field. The absorption spectrum would thus mimic the photon distribution function of the driving field, which can therefore be recovered from the spectrum. The condition under which the photon distribution function of the driving field could be recovered is that the associated vacuum Rabi frequency should be larger than the atomic decay rates. Method to determine the photon statistics based on Autler-Townes spectroscopy is closely related to the one discussed in this paper. Another closely related scheme which has been experimentally realized for the determination of photon statistics is that of quantum Rabi oscillation. Rabi oscillations have been observed in vacuum and in small coherent fields. Its Fourier components show the discrete nature of field and the weighted Fourier components yield the photon number distribution in the field.

Scheme

We consider a system of three-level atoms initially in the ground state interacting with a quantized radiation field inside a cavity. The quantized cavity field drives the upper levels of the atom. We are interested in finding the photon statistics of the field. This is done by probing the absorption spectrum. The atoms are prepared initially in the ground state. We show that the photon statistics of the radiation field inside the cavity can be determined by looking at the absorption spectrum. Under exact resonance both the real and imaginary parts of the susceptibility vanish and the medium becomes transparent. This result is valid for arbitrary photon statistics of the driving field. An important and interesting fact is that the height of the peaks is independent of the excitation number n.

Including the contributions from all the photon excitations in the photon distribution function and in the limit that the decay rate much less than the vacuum Rabi frequency of the driving field, we get the complete absorption spectrum. This absorption spectrum will mimic the photon distribution function. We next illustrate our results by considering the example of a Schrodinger-cat state, which is a coherent superposition of two coherent states. The photon distribution is thus an oscillatory function of n. These oscillations are manifestation of non-classical features of the quantum statistics. The photon distribution function is plotted against n. The corresponding absorption spectrum is plotted. The photon distribution function, recovered from the absorption spectrum in the same way as mentioned above.

This scheme for the measurement of photon statistics through EIT is feasible within the presently accessible experimental limits. A small Fabry-Perot cavity, as reported by Hood where a single atom interacts with a cavity field, is appropriate for our scheme. Here g=60 times 2pi MHz, which is determined by the cavity geometry, and the atomic decay rate gamma=2.6 times 2pi MHz. These values are in accordance with the condition required by our scheme that g>> gamma in order to resolve the peaks of the photon distribution clearly. An improvement would be required as far as the cavity interaction time is concerned, which is small in this case, in contrary to our requirement. In the microwave region, however, a large cavity interaction time has been observed in addition to the desired values of g and gamma. One discrepency, however, has not been encountered here; the values of g and gamma referred here are for the same two levels, which is not our case.

Results

In this paper we have discussed a method based on absorption spectrum to measure the photon statistics of the radiation field using electromagnetically induced transparency. This is a conceptually simple and direct method and involves no cumbersome numerical inversions like that used in some other schemes for the same purpose. Another advantage of this method is that it is insensitive to the detector efficiency, which can create serious problems in the observation of nonclassical features of the quantum states.

Paper # 6

Atom Localization via Resonance Fluorescence

Introduction

In recent years, several schemes have been considered for the localization of an atom using the standing optical light field. These schemes are based on the possibilities of measuring the phase shift of either the standing wave or the atomic dipole caused by the passage of the atom through the field. We can also correlate a spatially varying potential with an atomic resonance frequency and the position distribution of the atoms. In these methods the position information is available only after the atom has passed through the field. However for many potential applications, it is desirable to obtain position information of the atom during its passage through the standing wave.

In this letter, we suggest a simple scheme to localize an atom inside the standing wave during its motion. This scheme utilizes the idea that the frequency of the spontaneously emitted photon carries the information about the position of the atom due to position-dependent Rabi frequency of the driving field. Therefore an atom is localized as soon as the spontaneously emitted photon is detected. This scheme presents a simple method for the localization of an atom using simple two-level atom interacting with the classical standing-wave field. In the presence of the driving field, dynamic Stark splitting of the atomic levels takes place and we get a three-peak resonance fluorescence spectrum. The splitting is directly proportional to the position dependent Rabi frequency. Our scheme exploits this fact and by measuring the frequency of the spontaneously emitted photon we can localize the atom during its motion through the standing wave field. It is worthwhile to mention that such a scheme, along with a similar scheme for atom localization based on Autler-Townes spectroscopy, affords a direct method to obtain information about the quantum state of the radiation field without any major numerical computations.

Scheme

We consider a two-level atom with a center-of-mass wavefunction f(x). The atom is moving along z-axis and interacts with a resonant standing-wave light field of wave-

vector "kappa" aligned along the x direction. The velocity component of the atom along z-axis is considered large enough so that the motion in this direction is treated classically. The driven atom radiates spontaneously and one of the modes of the scattered light interacts with the detector atom, initially in its ground state. We assume that the scattered light is absorbed by the detector atom and is excited to an appropriate energy level. Our aim is to find the conditional position distribution of the atom. We assume that the center-of-mass momentum of the atom A along x -axis does not change appreciably during its passage through the standing-wave. We can then neglect the kinetic energy term for the atom in the Raman-Nath approximation. Our scheme utilizes the fact that the frequency of the spontaneously emitted photon is directly related to the x dependent Rabi frequency of the driving field. The spectrum of the spontaneously emitted photons or scattered light mimics the position probability of the center-of-mass motion of an atom. The filter function here is directly proportional to the excitation probability of the detector atom. The problem therefore reduces to finding the excitation probability for a single photon detection. The detector atom is interacting with the scattered light due to the decay of atom A. The probability of exciting the detector atom is found by calculating the expectation value of the projection operator. This excitation probability is therefore proportional to the power spectrum of the scattered light emitted from the atom. In the steady state the field emitted by the atom is statistically stationary.

Results and discussions

In the resonance fluorescence spectrum we have three peaks. In our scheme of localization of an atom we replace the Rabi frequency with the position-dependent Rabi frequency. The peaks are now position dependent. Atom now undergoes different Rabi oscillation at different position in a standing wave and we get maxima in the position distribution corresponding to these Rabi frequencies. We show a three-dimensional plot of the conditional position distribution for an initially broad wave packet, as a function of the normalized position and detuning. We note that for zero detuning there is a uniform position probability distribution over the wavelength domain of the standing wave. This is due to the fact that the atom exhibits a peak at Delta=0 for any value of Rabi frequency. The heights of the peaks for all values of position are the same and we therefore obtain a uniform position distribution. Thus the conditional position distribution provides no information about the atom localization for Delta=0. An increase in detuning corresponds to the localization of the atom at different positions inside the standing wave depending on the value of the position-dependent Rabi frequency. We obtain four maxima of same heights and widths. For small values of Delta, these maxima are located near the nodes of the standing wave. However with the increased detuning, these peaks move towards the antinodes of the standing wave. There are no resonances for Delta >2G and we obtain a flat position distribution over the wavelength domain. These results indicate a strong correlation between the detuning of the scattered light and the position of the atom. The measurement of a particular frequency corresponds to the localization of the atom in a subwavelength domain of the standing wave.

A clearer picture of the dependence of localization scheme of atom on the position dependent Rabi frequency and detuning is demonstrated, where we show 2-dimensional plots of the conditional position distribution as a function of normalized

position for four different values of detuning. It is clear from these plots that the best resolved peak is obtained at Delta=G for which the signal to background ratio is maximum. We get a partial overlap of the adjacent peaks for the ranges 0< Delta <G and G<Delta<2G. This causes an enhancement of the background. The strength of these overlap and consequently the signal to background ratio, depends on how much the detuning deviates from the maximum value of G. However a complete overlap is observed for Delta=0 and 2G, which corresponds to the node and antinode, respectively.

We also investigate the dependence of the width of the best resolved peaks, for which the signal to background ratio is maximum. It is noted that the width decreases with the increase in the amplitude G of the position dependent Rabi frequency. Here we like to mention it again that the above power spectrum gives the conditional position distribution. The frequency of the spontaneously emitted photon is related to the detuning parameter. Hence the detection of the spontaneously emitted photon gives the immediate information about the position of the atom inside the optical field. Although the spontaneous emission process is isotropic in nature and would require using 4pi detectors in principle but for practical purposes, it is not necessary to measure every atom. It would be sufficient to detect only those atoms whose spontaneously emitted photon is certainly detected.

Paper # 7

Quantum state measurement of an entangled state via Ramsey interferometry

Introduction

A pure state of a pair of quantum system is called entangled if they do not factorize, that is, if each separate system does not have a pure state of its own. In other words we can say that the two systems are correlated in an entangled state. A mixed state is entangled if it cannot be represented as a mixture of factorizable pure states. During last many years a great deal of work has been devoted in order to highlight the significance of entanglement, particularly for mixed state of a bipartite system. Entangled state of two or more particles, which are specially, separated give rise to quantum phenomena that cannot be explained in classical terms that is why the quantum entanglement lies at the heart of the profound difference between quantum mechanics and classical physics. The previous work devoted to the measurement of a quantum system was concentrated on the single mode field in high-Q cavities. The quantum state of both the single-mode and multi-mode radiation field is completely described by the state vector for a pure state and by the density operator for a mixed state. Equivalent descriptions of the quantum state can be formulated in terms of the quasiprobability distributions such as P-representation, Q-representation, or Wigner distribution function. The reconstruction of full information of the quantum state of a given field is one of the most fundamental problems of the quantum state measurement. Generally the quantum state in a single experiment cannot be measured precisely. However, one needs to perform different experiments on identically prepared objects to infer the quantum state

from the recorded statistical distributions. This idea was experimentally verified in a quantum optical system proposed by Vogel and Risken. In their scheme, homodyne detection method was employed to measure the quadrature distributions of equally prepared light pulses. The Wigner function was then reconstructed using the said quadrature distributions. The field distribution function can also be measured by using the other methods such as separated oscillatory field, atomic beam deflection, the conditional measurement on atoms in a micromaser setup, and so on.

We are interested in the reconstruction of the Wigner function of entangled state present in two separate cavities. Here we show entangled state of photons in spatially separated cavities can be measured using the techniques of cavity quantum electrodynamics. For this purpose we used the idea that quantum state of radiation field can also be measured by measuring depressive phase shift produced by the off resonant cavities on the interacting atoms. In Ramsey type setup the phase shift produced in the two level interacting atoms is directly proportional with the number of photons present in the cavity. We displace the photon statistics of the entangled field by injecting the same frequency coherent state. The displaced photon statistics is then used to reconstruct the Wigner function of cavities in a straightforward manner.

Wigner Function of two mode entangled field state

Here we show that how we have reconstructed the Wigner function of the two entangled cavities. We start with the definition of the Wigner function described in. In this reference Cahill and Glauber had suggested that the Wigner function of the single mode field could be found by displacing the photon statistics of the cavity field. The measurement of the displaced photon statistics of the cavity field then leads for the reconstruction of Wigner function. The displacement in the cavity field is obtained by injecting a coherent state alpha inside it. Cahill and Glauber had obtained a expression for the Wigner function in terms of displaced photon statistics of the cavity field. The Wigner function of the cavity field can be found directly if the displaced photon statistics is known. We extend the idea of Cahill and Glauber to multimode entangle systems. We realize that the definition of the Wigner function, which belongs to a general class of phase-space distribution, can be written for the case of two-mode field also. Here we assume that there is a joint probability of m photons in cavity mode A and n photons in cavity mode B. As we need to measure the two separate entangled cavities that is why we use two coherent sources for the displacement of the states inside the cavities. The two separate sources are connected to the cavities such that coherent states are injected to cavity A and cavity B, respectively. Thus the state of the field inside the cavities is displaced. Thus the Wigner function of the field can be found directly if the displaced photon statistics is known for all values of alpha and beta.

Reconstruction of photon statistics of entangled state

Let us consider an entangled field inside two high-Q cavities. The presented scheme for the reconstruction of photon statistics of the entangled field state can be used for the reconstruction of full information in the form of Wigner function. We propose Ramsey type setup for the measurement of photon statistics of the entangled state. A two

level atom interacts first with Ramsey zone, the cavity A, the cavity B and in the last with the second Ramsey zone. The atoms are resonant with the Ramsey zones and are off-resonant with the cavity fields. The first Ramsey zone prepare the atoms in the superposition of their internal states while the two cavities A and B introduces a phase shift in the atomic states proportional to the total number of photons in the two cavities where as the second Ramsey zone again prepares the atoms in superposition of atomic states. At the output we measure the internal state of the atoms.

For the reconstruction of Wigner function of entangled state we propose to displace each cavity mode by two independent microwave sources resonant to cavity mode A and B, respectively. The microwave sources inject the coherent states in cavity A and B, respectively. This injection of the coherent states inside the cavities displaces the state of the two separate entangled cavities.

We take the two-level atoms that are initially prepared in the excited state by laser excitation before the interaction with the first Ramsey zone. The atoms in excited state are then passed through the first Ramsey zone. The interaction time of the atoms with the first Ramsey zone is adjusted such that they see a pi/2 pulse, this causes the preparation of the atoms in the superposition of their internal states. Here we choose the relative phase of the atom and field equal to pi/2. After interacting with the first Ramsey zone the atoms pass through the first cavity A that is off resonant with the atomic transition frequency. The atoms go under a phase shift during their passage through the cavity due to dispersive atom field coupling. Brune discusses this dispersive type of atom field coupling in detail. The phase shift produced in the atomic states after interacting with the field present in the off-resonant cavity A depends on the number of photons inside it. During this interaction the atoms go under a phase change only and there is no gain or loss of photons between the atom and the cavity. The emerging atoms then interact with the second cavity B and pick another phase proportional to the number of photons in this cavity. The atoms, in the last, interact with the second Ramsey zone. Here the atom again sees a pi/2 pulse and the relative phase of the atomic probe and the Remsey field is 3pi/2.

The complete atom-field state is entangled and is complicated at this stage, it is however clear that a measurement of the atoms in state a or b reduces the entanglement of atoms with the field inside the cavities. At the output we measure the atoms in the exited or ground state with the help of field ionization detectors. The probability of the atoms to be detected in excited state can be found by taking the trace. Experimentally we get the probability in excited state by the detection of the atoms in state a over the ensemble of the detections on the identically prepared systems. Now at this stage we realize that the difference of the probabilities of the excited and ground state can be used to find the Wigner function of the entangled cavities. This gives us the method for finding the Wigner function of the entangled cavities.

It is clear that the Wigner function of the entangled cavities can directly be found with the knowledge of the difference of the probabilities of the excited and ground states of the atoms. The final states are the periodic functions, which exhibit a characteristic pattern of fringes. The frequency of the patterns depends on the interaction time of the atoms with fields. We take the initial distribution of the photons in the two cavities. The first atom passing through the cavity alters the photon statistics of the cavity field due to the back action of the measurement. The photon number distribution is multiplied by a oscillatory function on n.

Conclusion

In conclusion we have proposed a measurement technique for obtaining the full information of entangled state in two separate cavities. The scheme is based upon the Ramsey type setup in the separated oscillatory fields method. The Wigner function is obtained by the displaced photon statistics of the cavity fields. The two separate electromagnetic oscillators obtain this displacement of the field modes in two separate cavities.

Paper # 8

Quantum state measurement using phase-sensitive amplification in a driven three-level atomic system

Introduction

Quantum state measurement problem has attracted a lot of interest in recent years in the field of quantum optics. A number of schemes have been proposed to measure the non-classical states of the radiation field. These models incorporate the techniques based upon absorption and emission spectroscopy, dispersive atom-field coupling, the conditional measurement of the atom in a micromaser and others. As all the knowable information of the density matrix of a quantum system are contained in its Wigner function, so the knowledge of the Wigner function reveals the complete quantum state of the system. This scheme has been applied successfully to experimentally measure the vacuum and the squeezed states of the radiation field.

However, the finite efficiency of the photo detectors poses a series difficulty in the measurement of the quantum state. The quantum states are highly sensitive to the noise associated with the detectors inefficiencies. Inspite of some initial success, it is still beyond the scope of the current experiments to resolve the fine details of a quantum state being characteristic of non-classical behavior. For example, the oscillations in the photon number statistics for squeezed states have not been observed yet. In some recent studies it is shown that the measured probability distribution function becomes smoothed due to the finite detection efficiency. For example, instead of Wigner distribution function, smoothed quasiprobability distribution functions are reconstructed. In particular, for an overall 50 detector efficiency, Q function instead of Wigner function is reconstructed. An important question in this regard is how to overcome the finite efficiency of the detectors. In this paper, we present a model for the quantum state measurement using a two-photon amplification by three-level atoms in the cascade configuration, where coherence is induced between the top and the bottom levels by driving the atoms continuously with a strong external field. It was shown by Ansari, Banachloche and Zubairy that this system exhibits remarkable differences with the system where the atomic coherence is obtained before the interaction, so that there is no external driving field present during the amplification process. These two methods for generating the coherence are not equivalent. In fact it was found that the system with the driving field never reduces to the

one studied in Ref. Instead it exhibits, as a function of the driving field, a range of behavior, from a phase-insensitive amplifier for low driving field to an ideal parametric amplifier at the other extreme. Here in this study, it is shown that this system could be used for the reconstruction of the quantum state. In case of zero-detuning, for the large Rabi frequency, when noise in both the quadratures reduces to zero, the amplifier becomes identical to an ideal parametric amplifier and it successfully recovers the original quantum state.

The Wigner function of the quantum state can be obtained by calculating the complete distribution function for the quadrature values. The maximum amplification with reduced noise for different quadrature phases is obtained by driving the system for different values of the phase of the classical field. We have calculated the distribution function for an arbitrary quantum state after its amplification through a phase sensitive linear amplifier in a driven three-level atomic system. The distribution function of the noise free quadrature is then used to reconstruct the quantum state of the field using inverse Radon transformation well known in quantum tomography. We apply this model to a Schrodinger-cat state and discuss its reconstruction after its amplification through a two-photon phase sensitive linear amplifier in the zero-detuning limit. This model enables us to overcome the problems arising due to the finite efficiency of the detectors in homodyne measurement scheme. In an earlier paper, we proposed a model for the quantum state measurement using two-photon CEL.

Model for the Field-Density Matrix

Our amplifier consists of three-level atoms in cascade configuration. The transitions a-b and b-c are dipole allowed whereas, the transition a-c is dipole forbidden. We assume that the transition a-c may be induced by employing a sufficiently strong resonant external driving field. We are considering the linear amplifier, therefore, we treat a-b and b-c transitions quantum mechanically up to the second order in the coupling constant and a-c transition semi-classically to all the orders. We assume that the atoms are initially pumped incoherently to the upper level. For simplicity, the decay rate is considered to be same for all the three levels. The coherence is produced by the classical driving field and is responsible for the phase-sensitivity in the system. In the remaining calculation, we assume that the two-photon resonance condition is applicable. Here we consider a two-photon linear amplifier, and calculate the gain contribution for the field quadratures. It is interesting to note that G depends upon the phase of the classical driving field and the phase of the generalized quadrature. In the forthcoming subsection, we discuss the reconstruction of the Wigner function in the zero-detuning limit.

Reconstruction of the Wigner distribution function in the zero detuning limit

The zero detuning condition requires that the level b lies exactly in between the upper level a and the ground state level c. The time evolution of the Wigner function of the field can be evaluated by writing in terms of its Fokker-Planck equation for the Wigner distribution and by finding its time dependant solution. We find the evolution of the Wigner function for any arbitrary initial field. We are interested in the measurement of the quadrature distribution when the initial quantum state is amplified through a phase-

sensitive three level atomic system. In a balanced homodyne detection measurement scheme, the quadrature phase is characterized by the phase of the local oscillator. A complete distribution for the quadrature component is determined by scanning the field quadrature over a range of phase. Such distributions have recently been measured in quantum optical tomography.

The parametric limit of the amplifier requires that psi=0. This condition requires that for the measurement of the field quadrature with maximum gain in each phase, the amplifier atoms have to be prepared in a specific phase correspondingly. The control over the amplifier phase is quite subtle in our model and it is exercised by introducing the variation in the phase of the driving field, from out side the cavity. This makes this model more realistic in its application for the quantum state measurement problem. In the limiting case the Rabi frequency of the classical driving field is much larger than the atomic level width gamma, the expression for the gain parameter G reduces. In this condition the noise in both the conjugate quadrature reduces to zero and the amplifier becomes identical to a degenerate parametric amplifier. Once the quadrature distribution of the amplified quantum state is measured in balanced homodyne measurement, then the corresponding Wigner function can be reconstructed by carrying out the inverse Radon transformation familiar in optical tomographic imaging.

Reconstruction of the Wigner Function in Non-Zero Detuning Limit

A more general case for two photon resonance condition is that of non-zero detuning. In this case, in order to separate out the noise and the gain terms, we impose the additional conditions. In the non-zero detuning the generalised quadrature distribution again emerges out the same as except for the change in the gain parameter G and the parameter xi. In this case G would always be greater than one for all the values of Omega and gamma. However, excess noise would be induced in both the conjugate quadratures and the quantum information carried away by the field quadrature would start fading away. Therefore, the original state would be recovered only upto a particular order of Delta / gamma. It is clear that for Omega / Gamma $\rightarrow$ infty, and alpha t $\rightarrow$ infty, we recover the Wigner function for the original Schrodinger cat state. In the forth coming section, we present the results of our numerical simulations.

Results and discussion

We present the results of our numerical simulation. The plots of Wigner function for xi_0=2, alpha t=1 and Omega / Gamma =1,15 and 30, respectively for zero detuning case. The results shows that for Omega / Gamma =1 (phase-insensitive amplifier) the well known oscilltory behaviour of the Wigner function vanishes. However, with the increase in Omega / Gamma the oscillations start appearing. For Omega / Gamma =30, the original Wigner function is almost fully recovered. We present the plots of Wigner function for xi _0=2, alpha t=5 and Omega / Gamma =1,15,30 and 60. The complete Wigner function is obtained for Omega / Gamma =60, which is quite interesting. We also present the plots of Wigner function for xi_0=2, alpha t=10 and Omega / Gamma =1,30, 60 and 90. This shows that an increase in alpha t requires larger value of Omega / Gamma for complete reconstruction of the original quantum state. We show the plots of

Wigner function for alpha t=1 and Delta / Gamma =1,2,4 and 8. It is interesting to note that with the increase in detuning, the original quantum state is fully recovered. This scheme allows to overcome the problem of detector efficiency.

In conclusion, it is shown that in a three level atomic system where the atomic coherence is established by driving the atoms continuously through a strong external classical field allow us to fully recover the Wigner function of the initial quantum state.

Paper # 9

Wigner function reconstruction using time-dependent physical spectrum

Introduction

Quantum objects cannot be seen as they are. This is due to the fact that quantum mechanics does not permit us to observe a single physical object completely. If one of the conjugate variables, for example, position of the physical object is predicted precisely the other conjugate variable. However, statistically, the physical properties of identically prepared objects are reproducible and we can obtain a state or wavefunction, which describes an ensemble of these physical objects. The measurement of such a quantum state is of great importance due to the fact that it brings into light various fundamental laws of nature, which can be described only by quantum mechanics principles. The state or wavefunction of a physical system contains the complete information about the system and any future prediction can be inferred from the state or the wavefunction. The dynamics of a single mode light field is very much analogous to the dynamics of a quantum particle and that is why it gets considerable attraction amongst the researchers to measure the quantum state of the light. The other main reason of attraction in this area is due to the fact that several states of the radiation field exhibit non-classical features. These include the squeezed state and Schrodinger-cat state. The main question, however, is how the state of the quantum field can be measured?

In the last decade, an extensive effort is being made to measure the quantum state of the radiation field by using various indirect methods. These include techniques based on Ramsey method of separated oscillatory fields, atomic beam deflection, the quantum state tomography, quantum state endoscopy, and several others. Quantum state representations are P-, Q-representation, or the Wigner distribution function. These quasiprobability distributions have phase space representation and therefore, can be used for the evaluation of symmetrically ordered correlation functions of the creation and annihilation operators of the field. Most importantly the non-classical features of squeezed state and Schrodinger-cat state can be manifested in the quasiprobability distributions such as P-representation and the Wigner distribution function. Among these various quasiprobability distributions the Wigner function is of particular interest. It has a one to one correspondence with the state or wavefunction and it can take on negative values. This quasiprobability distribution is therefore, closely related to the state or wavefunction and it represents the state of a quantum system in phase space. It is possible

to reconstruct the Wigner function from other probability distributions that can be measured experimentally for an ensemble of identically prepared quantum systems.

In this article we present a scheme for reconstruction of the Wigner function using the two-time correlation function of atomic dipole operators. The system uses the Autler-Townes spectrum for a quantized radiation field in which the peaks are located at each number of photon. The associated Rabi frequencies are distributed according to the photon distribution function of the field and the photon distribution is recovered from the spontaneous emission spectrum in a straightforward manner. Here we consider the definition of the physical spectrum, which is proportional to the two-time correlation function, to calculate the spontaneous emission spectrum.

Model

In the present scheme we consider a system of three-level atoms with energy levels a, b and c passing through a quantized radiation field present inside the cavity. A source that injects a coherent state is connected to the cavity. The injected coherent state displaces the state of the cavity. The photon statistics of displaced state of the quantum field is therefore, represented by and the corresponding Wigner function. If we have the photon statistics for all values of alpha we can easily reconstruct the Wigner function. It is mentioned earlier that the photon statistics of the driving field can be found directly from the spontaneous emission spectrum. Therefore, our requirement now is to calculate the spontaneous emission spectrum of our three-level system. We consider that the atomic transition between level a and c is resonant with the cavity field and atom decay from the level a to b at the rate gamma. We also assume that the decay rates from the levels c and b are very small as compared to gamma and we may ignore these decays. The atom undergoes Rabi oscillation due to the interaction with the resonant cavity field and with a certain probability decays spontaneously to the energy level b. The time-dependent physical spectrum of the scattered light at some suitably chosen point in the far field is obtained by taking the Fourier transform of the normally ordered correlation function of the field.

Results and Discussion

We calculate the expression for the time-dependent spontaneous emission spectrum. The complete spontaneous emission spectrum consists of contribution from all the photon excitations in the photon distribution function. To reconstruct the Wigner function we inject a coherent state inside the cavity as discussed above. The spontaneous emission spectrum now depends on the complex amplitude. The peaks in this spectrum are located at the integral values, so, the only meaningful values are the integer values. We can now reconstruct the Wigner function in a straightforward way. It may be noted that the spontaneous emission spectrum for a given value of the injected field gives the Wigner function at a point alpha in the complex plane and we have to obtain spontaneous emission spectra for different values to reconstruct the complete Wigner function.

We plot the Wigner function of the Schrodinger-cat state. The oscillations at the center of the Wigner function in between the two Gaussian hills exhibits the interference structure due to the quantum superposition of both amplitudes. It is also clearly observed

that in certain regions of Wigner function becomes negative showing the non-classicality of the Schrodinger-cat state. We show the contour plot of the Schrodinger-cat state, which gives a beautiful picture of the contour lines in the Wigner function. We show the plots of the reconstructed Wigner function from the time-dependent spontaneous emission spectrum. The Figure shows a perfect recovery of the original Wigner function. The contour plot of the Wigner function shows the contour lines of the distribution, this is in perfect agreement with the original plot.

In summary, in this article we have suggested a scheme to reconstruct the quantum state of the radiation field inside the cavity. This scheme uses a time-dependent physical spectrum, which is more realistic approach as far as the spontaneous emission spectrum measurement is concerned. In present scheme we do not require the atoms to be in coherent superposition of states, which is very difficult to attain, as it is a highly unstable state. A major advantage of this scheme is that it is directly applicable for the measurement of a quantum state and again no cumbersome mathematical manipulations are required for this purpose. Another advantage of the proposed scheme is that it is strongly independent of the detector efficiency. As we have used the spectrum for the calculation of the quantum state, a few no-photon counts can easily be ignored. The condition that has to be satisfied in this scheme is that the ratio of gamma / Omega, has to be very small in order to keep the peaks, resolved clearly. We have kept gamma / Omega =0.01. This small ratio of gamma and Omega has been achieved experimentally in the microwave and optical regions, respectively. The definition used for the spontaneous emission spectrum is more phenomenogical rather than the physical. The realistic approach for calculating the spontaneous emission spectrum is to use the two-time correlation function of the atomic dipole operators.

Paper # 10

Measurement of entangled state via atomic beam deflection in Bragg's regime

Introduction

In twentieth century the quantum theory of physics has been a fascinating playground to study the nature of electromagnetic radiations and matter. In this subject, the forces on atom by light have received much theoretical and experimental attention during past many years, not only because of interest in the basic atom field interaction, but also for the measurement of an unknown state of electromagnetic field which poses an interesting question in it. The measurement of the cavity field had gained a very high attention because of the possibility of the quantum computers, quantum teleportation, quantum cryptography, dense coding and many more. Generally the quantum state in a single experiment cannot be measured precisely. However, one needs to perform different experiments on identically prepared objects to infer the quantum state from the recorded statistical distributions. This idea was experimentally verified in a quantum optical system proposed by Vogel and Risken. In their scheme, homodyne detection method was employed to measure the quadrature distributions of equally prepared light

pulses. The Wigner function was then reconstructed using the said quadrature distributions. The other methods for the measurement of cavity field are based on dispersive atom-field coupling in a Ramsey method of separated oscillatory fields, atomic beam deflection, and absorption and emission spectrum. An excellent review of quantum state reconstruction is given in. Most of this work was devoted to the measurement of a single mode field in high-Q cavities.

The measuring process, in general, depletes the field if usual photon-counting technique is employed. For an ideal measurement thus we require that the system probe does not feed back noise into the variable that are being measured. This type of measurements is referred as quantum non-demolition measurements. The schemes of quantum non-demolition measurement may be based on, dispersive atom-field coupling in Ramsey type setup, optical Kerr effect, or atomic scattering. Among these schemes the atomic scattering method provides a nice tool for the measurement of field photon statistics, as the momentum distribution of deflected atoms is a function of field photon number The atomic diffraction from the electromagnetic field may be divided into two regimes, one in which the recoil energy of the field is much grater then the Rabi frequency (the Bragg regime) and the other in which the recoil energy is much less then the Rabi frequency (Raman-Nath regime). The theory of Bragg diffraction was given by Bernhardt and Shore and was reported in several experiments in which up to 8th order of diffraction has been observed. All these experiments were done with the classical field in cavities, however the advancements in technology, have made it possible to realize the diffraction of atoms from quantized cavity field. This paper deals with the utilization of atomic scattering in Bragg regime for quantum non-demolition measurement of joint photon statistics of entangled state in two separate cavities.

There are few schemes reported for the reconstruction of multi-mode field inside a cavity. A scheme for the reconstruction of entangled state in a cavity has been put forwarded by Kim and Agarwal. They used the idea that probability of atomic inversion after a two-level atom interacts with a cavity field is directly related to the Wigner characteristic function. Ikram and Zubairy proposed another scheme based on Autler-Townes spectroscopy to reconstruct the two-mode entangled state in a high-Q cavity. The scheme by Davidovich uses Ramsey type setup to reconstruct the GHZ state in three particles. Having a well justified and mathematically tractable measure of entanglement is likely to be value in a number of areas of research including the study of decoherence in quantum computers and the evaluation of quantum cryptographic schemes.

Here we are interested in the reconstruction of the Wigner function of entangled state present in two separate cavities. We show that the entangled field states in spatially separated cavities can be measured using the techniques of Bragg's diffraction of two-level atoms from the entangled field. We study the deflection of atoms from far detuned high-Q cavities in first order Bragg regime and develop quantum non-demolition measurement scheme based on it. The method adopted here is the reduction of entangled state to a Fock state, and then reproduce the field state by repeated measurements. Our suggested scheme successfully reproduces the photon statistics of the entangled electromagnetic field in two separate cavities. The photon statistics of the entangled field state does not give the full information of the field state; rather it gives the information only about the diagonal matrix elements. To get the information about the off-diagonal matrix elements we propose to reconstruct the Wigner function of the entangled field

state to be measured. For this purpose we propose to displace the field state in two cavities by injecting coherent states to the cavities. The displaced photon statistics is then used to reconstruct the Wigner function of cavities in a straightforward manner.

Energy-momentum conservation argument

In optical Bragg scattering, the condition for constructive interference of atomic de Broglie waves requires that the angle of incidence to the standing wave plane must be the one of the nth order scattering angles that satisfies Bragg relation. Similarly, the Bragg diffraction of a well-collimated atomic beam by the planes of standing wave electromagnetic field can be viewed as the deflection of the atomic de Broglie waves from standing wave of the field. It is important to note that no deflection occurs if the angle of incidence does not correspond to one of the scattering orders. This results in momentum transfer only for discrete initial values of atomic momentum (i.e., the only process which can conserve both the energy and momentum is the scattering in which the incidence angle satisfies the Bragg's condition). The situation of atomic scattering is similar to the Bragg diffraction in X-ray scattering from crystals, and dictates that by changing the longitudinal component of momentum we can change the order of Bragg scattering. It is also clear that the atom-field interaction in the cavity reverses the direction of motion (in longitudinal direction) of the atoms only. The magnitude of the momentum remains same before and after the interaction, thus the energy momentum is conserved in Bragg regime.

Scheme for quantum nondemolition measurement of entangled field

We consider an a two mode entangled field state present in two separate cavities. The state vector of the entangled field contains the joint probability of having m_{A} photons in cavity A and m_{B} photons in cavity B. We present a simple scheme for the measurement of this type of entangled field states. A two-level atom interacts off-resonantly with the standing wave fields in the cavities in cascade fashion. We take the atom having detunning between the atomic transition frequency and the frequency of the cavity mode. The atom first interacts with the field present in the cavity A, then it interacts on his way with the second cavity B. The probe in this scheme is the momentum states of the atoms in Bragg regime. In this regime, it is assumed, that the momentum component of the atoms along transverse direction is very large, so one can treat it classically. At the same time, the atoms have well-defined momentum states in the direction of wave propagation i.e., the longitudinal component of the atomic momentum is well defined, and we can treat it quantum mechanically. The atoms are prepared in the ground state g, (which ensure us the quantum non-demolition measurement) with initial momentum state in the direction of wave propagation. After interacting with two cavities the atoms are detected in either of the two momentum states with the velocity selective detectors placed after the cavity B. We suggest to keep the injection rate of the atoms very low so that there is only one atom interacting with the cavity mode at a time. In Hamiltonian we neglect the constant momentum components along the transverse direction as discussed earlier. We first specialize to the atom-field interaction in cavity A.

The conditions under which the proposed scheme acts as a quantum nondemolition measurement can be obtained by taking the case when the detuning of the atom-field is very large as compared to the recoil energy. Under this condition we can ignore the recoil energy term in comparison with the detunning term. Here it is clear that the above limit also prevents the atom in going to excited state, thus the decoherence effect by spontaneous emission is not present. In Bragg's regime the atom may either go undeflected or it may get deflected. The angle of the deflection depends upon the number of interactions with the cavity field. For depressive atom-field interaction, the number of interactions of the atom with the field remains even, i.e., the atom goes through complete cycles of consecutive excitations and de-excitations. This is because of the large detunning limit, therefore during interaction with the field, the atom undergoes through a complete Rabi oscillation. Due to this the atom emerges from the cavity in the same initial ground state conserving energy, and with a total momentum change of lhk in the direction of wave propagation, (i.e., for each complete cycle the momentum transferred to the atom is zero or 2hk). This condition under which the energy of the cavity do not change is at the heart of the quantum nondemolition measurement process.

The difference between Bragg scattering and Naman-Nath scattering is that there is only one possible diffraction order in the former case, and in the lateral case diffraction in many orders is possible simultaneousiy. The reason for many possible diffraction orders is that in Raman-Nath scattering, the atomic deBrogli wave is very sharply focused which is contrary to Bragg scattering. Consequently, in Raman-Nath scattering the atom could scatter into many different orders still conserving energy and momentum and the diffraction pattern remains symmetric about initial atomic trajectory. The absorption and stimulated emission of photon pairs causes the change in the direction of momentum along the wave propagation in such a way that the magnitude and its kinetic energy remain unchanged. Hence the momentum vector lies on a circle of constant energy in momentum space. Now it is clear that the probabilities of atomic momenta in first order Bragg diffraction oscillates only between the two probabilities and outside this range acquire very less contribution. We apply adiabatic approximation, which dictates that the slow varying amplitudes dominate the time evolution. Thus, we obtain a closed set of equations for probability amplitudes, which can be decupled by differentiating and putting the values from the respective equations. The final probability amplitudes of the atom-field interaction now act as initial conditions for the interaction of atom with the cavity B. Again the evolution of the system is similar as it was in the first cavity with only one change of the initial momentum states as the two momentum components are possible for the atoms after interaction with the cavity A. These two probability amplitudes clearly display oscillations of the momentum states where as all other probability amplitudes remain insignificant as demanded by the adiabatic approximation. Thus knowing the probabilities of the momentum states by the repeatedly measurement of the momentum of the diffracted atoms we can determine the state of the cavity field.

As the probability of detecting the atoms in momentum state p_{0} or p_{-2} is the product of joint photon statistics and an oscillatory function, so the photon number distribution can be transformed after detecting the atom in any of the momentum state. The sinusoidally varying function shows that the argument of this function contains the information of the entangled field modes. The spacing of the fringes depends upon the number of photons present in the entangled cavities. The photon number for which the

fringe function is closest to zero, are efficiently decimated thus causing the reduction in the field photon statistics. The oscillatory nature of the probabilities for the momentum states p_{0} and p_{-2} is due to the quantum interference process.

If the process is repeated on the same field with an atomic beam having different velocities and the atoms are detected in any of the momentum state then photon number distribution is suppressed and finally we arrive at fixed photon numbers in two cavities. Let these numbers be r and s in cavity A and B, respectively. We repeat the same process until we get another set of photon number say r^{prime} and s^{prime} in two cavities. We continue the process until the field state in two cavities is finally projected on an intensity pattern, even though no energy has exchanged between the atoms and the field. This is at the heart of quantum non-demolition measurement.

This situation is completely analogous to the quantum non-demolition schemes discussed, where the field initially in a coherent state transforms to Fock state with the measuring sequence. In our case the information acquired by detecting a sequence of atoms also modifies the entangled field function step by step, until it eventually collapses into a Fock state. We show how the photon number distribution transformed to Fock state after detection of a sequence of atoms with momentum state p_{0} or p_{-2}. Such repeated measurements lead to the determination of the photon statistics of the cavity field. Note that in this scheme the spread in the vertical velocity of the atom does not require any control. On contrary, the dispersion in the atomic vertical velocity, which corresponds to the uncertainty in phase helps as it makes easy to determine the entangled state of the field.

Wigner Function of two mode entangled field state

The Wigner function of the field can be found directly if the displaced photon statistics is known for all values of alpha and beta. We have already seen that the photon statistics of the entangled field can be found by reducing the initial photon statistics to a Fock state and then repeating the experiment to a ensemble of the identical entangled cavities. In our scheme to get the Wigner function of the entangled state we propose to displace each mode by injecting coherent states alpha and beta into the cavity A and B, respectively. Experimentally coupling two resonant classical oscillators to the cavity modes A and B, respectively, carries out this operation. The displaced photon statistics of the entangled field allows us to calculate the Wigner function of them. In addition the momentum states for each value of alpha and beta gives the two-mode Wigner function in a straightforward way, which shows that the two-mode Wigner function of entangled field state can be found by the displaced state of the entangled cavities for all values of alpha and beta.

Result and Discussion

We want to do quantum non-demolition measurement of the entangled field in two separate cavities. For this purpose we have used atomic Bragg diffraction. Moreover, we take the large detuning between the entangled field frequency and the atomic transition frequency. This ensures that atom does not exit from the cavity in excited state and there is no spontaneous emission that contributes a photon in arbitrary direction. This

was noted that the atom comes out of the entangled cavities without altering the photon number of the field with two momentum states. Thus in this process quantum non-demolition is well maintained. The information of the field photon number can now be extracted from the momentum probability distribution of the deflected atoms. We have seen from our calculations that the momentum probability of the scattered atoms is a periodic function and the argument of the periodic function contains the field photon statistics. In order to find the entangled field state we have used the method of reducing it to a single Fock state and then to measure it. Interaction of each atom with the entangled field present in two separate cavities updates the cavities field statistics depending upon the interaction time. The photon statistics gets multiplied by oscillatory function, which has periodic maximas and minimas. The position of the minimas changes with the interaction time of atom with field. The interaction time of the atom can be controlled by controlling the transverse velocity of them. Each atom in different interaction time eliminates some photon numbers in the distribution, until after a few number of atoms only one photon number state is left, which then does not change. By repeating this simulation and counting the number of times each n is appearing, we reconstruct the original photon distribution.

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List of Publications

1. Sajid Qamar, Shi- Yao Zhu, and M. Suhail Zubairy, Phys. Rev. A 61, 63806 (2000).

2. Fazal Ghafoor, S. Y. Zhu, and M. S. Zubairy, Phys. Rev. A 62, 13811 (2000).

3. Manzoor Ikram, Shi-Yao Zhu, and M. Suhail Zubairy, Phys. Rev. A 62, 22307 (2000).

4. Mashhood Ahmad, Shahid Qamar and M. Suhail Zubairy, Phys. Rev. A 62, ?3814 (2000).

5. Mashhood Ahmad, Shahid Qamar and M. Suhail Zubairy, Phys. Rev. A 64, ?3811 (2001).

6. Tasnim Azim, Sajid Qamar, and M. Suhail Zubairy (accepted in J. Opt. B: Quantum & Semiclass. Opt. B)

7. Ashfaq Hussain Khosa and M. Suhail Zubairy, (to be submitted).

8. Ashfaq Hussain Khosa and M. Suhail Zubairy, (to be submitted).

9. Ashfaq Hussain Khosa and M. Suhail Zubairy, (to be submitted).

Ph. D. Completed

Mr. Ashfaq Hussain Khosa has completed the research work of his Ph. D. degree. He was enrolled in June 1998 and will be submitting his thesis soon.

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Atom localization via resonance fluorescence

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We propose a simple scheme of atom localization based on resonance fluorescence from a standing-wave field. The Rabi frequency is position dependent and therefore the spontaneously emitted photon carries the information of the atomic center-of-mass motion. This leads to atom localization even during the flight through the standing-wave field.

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Precision position measurement of atoms has a vast history of interest due to its involvement in many applications like laser cooling, Bose-Einstein condensation, and atom lithography. The experimental progress in using light forces to manipulate the motion of atoms [1] make it more desirable to get high resolution position measurement of atoms with optical techniques.

In recent years, several schemes have been considered for the localization of an atom using the standing optical light field. For example, Storey *et al.* [2] and Marte and Zoller [3] proposed the idea of a virtual "optical slit." In that scheme, the atom is localized by measuring the phase shift of the optical field in a cavity due to the spatially varying atom-field coupling. The localization in position space based on the phase-shift measurement on the field is further investigated via homodyne detection [4,5] by using the method of quantum trajectories [6]. A related technique for the position measurement of the atom is used by Kunze *et al.* [7] in which the phase shift of the atomic dipole, rather than the light field is used. Kien *et al.* further investigated this method and showed that a coherent cavity field substantially enhances the resolution as compared to a classical field [8]. In a recent experiment, Kunze *et al.* [13] demonstrated how the entanglement between the atom's position and its internal state allows one to localize the atom without *directly* affecting the particle's spatial wave function. They reported the possibility of producing narrow localization structures with widths below $\lambda/20$.

Other techniques such as atom imaging methods are proposed by Thomas and co-workers [9–11]. These methods are based on resonance imaging, i.e., a spatially varying potential shifts the resonance frequency of an atomic transition. Therefore the resonance frequency is position dependent and position distribution is determined by spectroscopic methods. They achieved a spatial resolution of 1.7 μm for the atomic position measurement by using the technique of atom imaging in high magnetic field gradients. Thomas *et al.* further demonstrated that a suboptical wavelength localization can be achieved by using light-shift gradient for atom imaging [12].

In this article we suggest a simple scheme to localize an atom inside the standing wave during its motion. This scheme utilizes the idea that the frequency of the spontaneously emitted photon carries the information about the position of the atom due to its dependence on the position dependent Rabi frequency of the driving field. Therefore an atom is localized as soon as the spontaneously emitted photon is detected. This scheme presents a simple method for the localization of an atom using a simple two-level atom interacting with the classical standing-wave field. In the presence of the driving field, dynamic Stark splitting of the atomic levels takes place and we get a three-peak resonance fluorescence spectrum. The splitting is directly proportional to the position-dependent Rabi frequency. Our scheme exploits this fact and by measuring the frequency of the spontaneously emitted photon we can localize the atom during its motion through the standing field. It is worthwhile to mention that such a scheme, along with a similar scheme for atom localization based on Autler-Townes spectroscopy [14], affords a direct method to obtain information about the quantum state of the radiation field without any major numerical computations [15,16].

We consider a two-level atom A with energy levels $|a\rangle$ and $|b\rangle$ and transition frequency ω_{ab} that is described by a center-of-mass wave function $f(x)$. The atom is moving along the z axis and interacts with a resonant standing-wave light field of wave vector $\kappa=\omega_{ab}/c$ aligned along the x direction as shown in Fig. 1. The velocity component of the atom along the z axis is considered large enough so that the motion in this direction is treated classically. The driven atom radiates spontaneously and one of the modes of the scattered light interacts with the detector atom B, initially in its ground state. The detector atom consists of the ground level $|\beta\rangle$ and a set of excited levels $|\alpha_k\rangle$. We assume that the scattered light of wave vector $\mathbf{k}_0$ is absorbed by the detector atom and is excited to an appropriate energy level $|\alpha_{k_0}\rangle$. Our aim is to find the conditional position distribution of the atom A, i.e., the conditional probability $W(x;t|\alpha_{k_0})$ of finding the atom A at position x at time t when the detector atom B is excited to the level $|\alpha_{k_0}\rangle$.

We assume that the center-of-mass momentum of the atom A along x axis does not change appreciably during its passage through the standing wave. We can then neglect the kinetic energy term for the atom in the Raman-Nath approximation. The interaction Hamiltonian for the atom A, in the dipole and rotating-wave approximations, is therefore given by

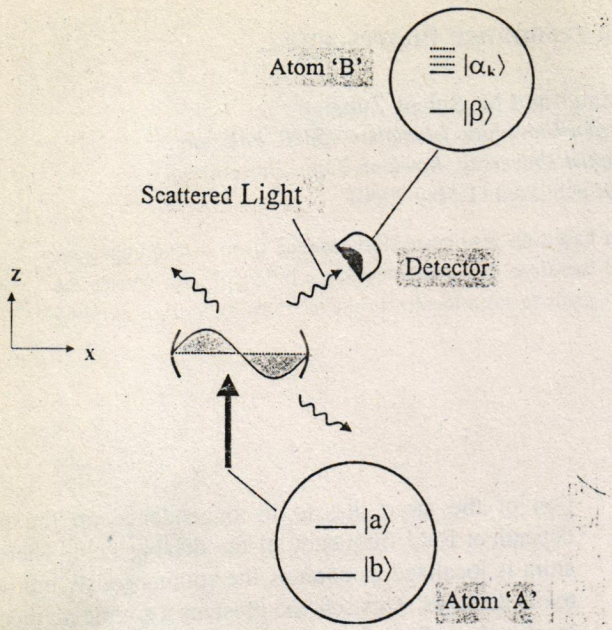

FIG. 1. Two-level atom A moving along the z axis and interacting with a resonant standing-wave light field of wave vector $\kappa = \omega_{ab}/c$ aligned along the x axis. The driven atom A radiates spontaneously in all directions. The detector atom B, consisting of the ground level $|\beta\rangle$ and a set of excited levels $|\alpha_k\rangle$, absorbs the emitted photon in mode $\mathbf{k}$.

$$H(t)=\hbar g(x)[|a\rangle\langle b|+|b\rangle\langle a|]+\hbar\sum_{\mathbf{k}}[g_{\mathbf{k}}(x)|a\rangle \times\langle b|e^{-i(\nu_k-\omega_{ab})t}b_{\mathbf{k}}+g^*_{\mathbf{k}}(x)|b\rangle\langle a|e^{i(\nu_k-\omega_{ab})t}b^\dagger_{\mathbf{k}}], \tag{1}$$

where $g(x)=G\sin(\kappa x)$ is the position-dependent Rabi frequency, the operators $b_{\mathbf{k}}$ and $b^\dagger_{\mathbf{k}}$ are the annihilation and creation operators for the photons in the reservoir modes with frequency $\nu_k=c|\mathbf{k}|$, and $g_{\mathbf{k}}(x)$ is the coupling constant between the atom and the vacuum mode $\mathbf{k}$. The state vector for the complete atom-field system is

$$|\Psi(t)\rangle=\int dx\, f(x)|x\rangle\sum_{n_{\mathbf{q}}}[C_{a,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta}(x;t)|a,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta\rangle + C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta}(x;t)|b,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta\rangle + C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)|b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}\rangle], \tag{2}$$

where $C_{i,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta}(x;t)$ is the position-dependent probability amplitude with the atom A being in the level $|i\rangle$ $(i=a,b)$ with no spontaneously emitted photon present in the mode $\mathbf{k}_0$ and n photons present in the mode $\mathbf{q}$, while the detector atom remains in the ground level $|\beta\rangle$. Similarly $C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)$ is the probability amplitude for the atom A to be in the level $|b\rangle$ after emitting one photon in the $\mathbf{k}_0$th mode and having n photons in the mode $\mathbf{q}$; the emitted photon is absorbed by the detector atom B exciting it to the state $|\alpha_{\mathbf{k}_0}\rangle$ with no photon left in the $\mathbf{k}_0$th mode.

Our scheme utilizes the fact that the frequency of the spontaneously emitted photon is directly related to the x-dependent Rabi frequency of the driving field. We now see how the spectrum of the spontaneously emitted photons or scattered light mimic the position probability of the center-of-mass motion of an atom. The conditional probability $W(x;t|\alpha_{\mathbf{k}_0})$ of finding the atom A at position x at time t when the detector atom excites to the level $|\alpha_{\mathbf{k}_0}\rangle$ is

$$W(x;t|\alpha_{\mathbf{k}_0})=\sum_{n_{\mathbf{q}}}|\langle x|\psi_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}\rangle|^2, \tag{3}$$

where

$$|\psi_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}\rangle=\mathcal{N}\langle\alpha_{\mathbf{k}_0}|\langle n_{\mathbf{q}}|\langle 0_{\mathbf{k}_0}|\langle b|\Psi(t)\rangle =\mathcal{N}\int dx\, f(x)C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)|x\rangle. \tag{4}$$

Here $\mathcal{N}$ is a normalization factor. Thus the conditional position probability is given by

$$W(x;t|\alpha_{\mathbf{k}_0})\equiv W(x)=|f(x)|^2P(\omega,x,t), \tag{5}$$

with $\omega=|\mathbf{k}_0|/c$ and

$$P(\omega,x,t)=|\mathcal{N}|^2\sum_{n_{\mathbf{q}}}|C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)|^2. \tag{6}$$

Here $P(\omega,x,t)$ is the filter function which is directly proportional to the excitation probability of the detector atom. The problem therefore reduces to finding the excitation probability $P(\omega,x,t)$ for a single photon detection.

The detector atom is interacting with the scattered light due to the decay of atom A. The interaction picture Hamiltonian for the interaction between the detector atom located at position vector $\mathbf{r}$ and the scattered field $E^-(\mathbf{r},t)$, in the rotating-wave approximation, is

$$H_d=-\sum_{\mathbf{k}}[\wp_{\alpha_{\mathbf{k}}\beta}\sigma_{\alpha_{\mathbf{k}}\beta}E^+(\mathbf{r},t)e^{i\omega t} +\wp^*_{\alpha_{\mathbf{k}}\beta}\sigma_{\beta\alpha_{\mathbf{k}}}E^-(\mathbf{r},t)e^{-i\omega t}]. \tag{7}$$

For the detector atom initially in its ground state $|\beta\rangle$ and the field in some state $|f\rangle$, the state of the atom-field system at time t is given by

$$|\Psi(t)\rangle=U_I(t)|\beta\rangle|f\rangle. \tag{8}$$

We then have

$$|\Psi(t)\rangle=\left[1-\frac{i}{\hbar}\int_{t_0}^{t}dt'H_d(t')\right]|\beta\rangle|f\rangle. \tag{9}$$

The probability of exciting the detector atom to level $\alpha_{\mathbf{k}_0}$ is found by calculating the expectation value of the projection operator $|\alpha_{\mathbf{k}_0}\rangle\langle\alpha_{\mathbf{k}_0}|$, i.e.,

$$P(\omega,x,t)=\langle\Psi(t)|\alpha_{k_0}\rangle\langle\alpha_{k_0}|\Psi(t)\rangle. \tag{10}$$

The resulting expression for the excitation probability is

$$P(\omega,x,T)=\frac{\wp^2}{\hbar^2}\int_{t_0}^{T}dt_1\int_{t_0}^{T}dt_2\langle E^-(\mathbf{r},t_1)E^+(\mathbf{r},t_2)\rangle \times e^{-i\omega(t_1-t_2)}. \tag{11}$$

This excitation probability $P(\omega,x,T)$ is therefore proportional to the power spectrum of the scattered light [17] emitted from the atom A. In the steady state ($T\gg\Gamma^{-1}$) the field emitted by the atom is statistically stationary, i.e., the field correlation function $\langle E^-(\mathbf{r},t_1)E^+(\mathbf{r},t_2)\rangle$ depends only on the time difference $\tau=t_1-t_2$. We then obtain

$$P(\omega,x,\infty)=\frac{1}{\pi}\mathrm{Re}\int_0^\infty d\tau\langle E^-(\mathbf{r};t)E^+(\mathbf{r};t+\tau)\rangle e^{+i\omega\tau}. \tag{12}$$

As the field operators $E^-(\mathbf{r},t)$, $E^+(\mathbf{r},t)$ are proportional to the atomic operators $\sigma_+(t)$, $\sigma_-(t)$, respectively, we obtain

$$\langle E^-(\mathbf{r};t)E^+(\mathbf{r};t+\tau)\rangle=I_0(\mathbf{r})\langle\sigma_+(t)\sigma_-(t+\tau)\rangle, \tag{13}$$

where $I_0(\mathbf{r})$ is a constant. The two-time correlation function of atomic dipole operator $\langle\sigma_+(t)\sigma_-(t+\tau)\rangle$ can be calculated by using the quantum regression theorem. It follows on taking the Fourier transform of $\langle\sigma_+(t)\sigma_-(t+\tau)\rangle$ that the power spectrum of fluorescence light is [18]

$$P(\omega,x,\infty)=\frac{I_0(r)}{4\pi}\left(\frac{4g^2(x)}{\Gamma^2+8g^2(x)}\right)\left[\frac{4\pi\Gamma^2}{\Gamma^2+8g^2(x)}\delta(\Delta)+\frac{\Gamma}{\Delta^2+(\Gamma/2)^2}+\frac{c_+}{(\Delta+\mu)^2+(3\Gamma/4)^2}+\frac{c_-}{(\Delta-\mu)^2+(3\Gamma/4)^2}\right], \tag{14}$$

where $\Delta=\omega_{ab}-\omega$, $\mu=\sqrt{4g^2(x)-\Gamma^2/16}$, and

$$c_\pm=\frac{3\Gamma}{4}\left[\frac{8g^2(x)-\Gamma^2}{8g^2(x)+\Gamma^2}\right]\pm(\Delta\pm\mu)\frac{\Gamma}{4\mu}\left[\frac{40g^2(x)-\Gamma^2}{8g^2(x)+\Gamma^2}\right]. \tag{15}$$

The expression for $P(\omega)$ simplifies considerably when the Rabi frequency is much larger than the decay rate of the atom i.e., $g(x)\gg\Gamma$. The resulting expression for the spectrum is

$$P(\omega,x,\infty)=\frac{I_0(r)}{8\pi}\left[\frac{3\Gamma/4}{[\Delta+2g(x)]^2+(3\Gamma/4)^2}+\frac{\Gamma}{\Delta^2+(\Gamma/2)^2}+\frac{3\Gamma/4}{[\Delta-2g(x)]^2+(3\Gamma/4)^2}\right]. \tag{16}$$

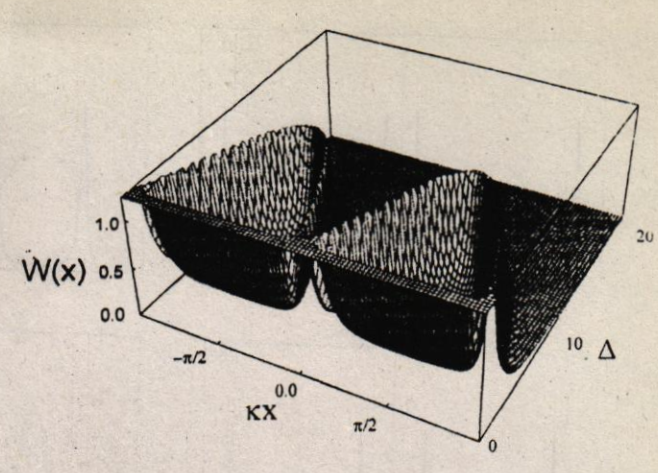

FIG. 2. Conditional position probability distribution $W(x)$ as a function of normalized position $\kappa x(0\leq\kappa x\leq 2\pi)$, and detuning Δ, for $g(x)\gg\Gamma$. For $\Delta=0$ there is a uniform position probability distribution over the wavelength domain of the standing wave. By an increase in Δ, maxima corresponding to atom localization at different positions inside the standing wave (initially near the nodes of the standing wave) for four different values of $g(x)$, are observed. These maxima move away from the nodes with increasing detuning. For $\Delta=\pm 2G$, four maxima merge into two and lie on the antinodes of the standing wave and for $|\Delta|>2G$ no resonances exist and a flat position distribution over the wavelength domain is obtained.

This is the well-known three-peak Mollow spectrum, the only difference being the position dependence of the Rabi frequency.

In the resonance fluorescence spectrum we have three peaks centered at $\Delta=0$ and $\Delta=\pm 2g$. In our scheme of localization of an atom we replace the Rabi frequency g with the position-dependent Rabi frequency $g(x)=G\sin(\kappa x)$. The peaks are now x dependent and are located at $\Delta=0$ and $\Delta=\pm 2G\sin(\kappa x)$. The atom now undergoes a different Rabi oscillation at a different position in a standing wave and we get maxima in the position distribution corresponding to these Rabi frequencies. In Fig. 2 we show a three-dimensional plot of the conditional position distribution $W(x)$ for an initially broad wave packet as a function of the normalized position κx and detuning Δ. We note that for zero detuning there is a uniform position probability distribution over the wavelength domain of the standing wave. This is due to the fact that the atom exhibits a peak at $\Delta=0$ for any value of Rabi frequency, and hence for all values of κx. The heights of the peaks for all values of position are the same and we therefore obtain a uniform position distribution. Thus the conditional position distribution provides no information about the atom localization for $\Delta=0$. An increase in detuning corresponds to the localization of the atom at different positions inside the standing wave, depending on the value of the position-dependent Rabi frequency $g(x)$. We obtain four maxima of same heights and widths in the region $0\leq\kappa x\leq 2\pi$ located at $\kappa x=\pm\sin^{-1}(\Delta/2G)\pm n\pi$ ($n=0,\pm 1$). For small values of Δ, these maxima are located near the nodes of the standing wave. However, with the increased detuning these peaks move towards the antinodes of the standing wave. For $\Delta=\pm 2G$, four maxima merge into two

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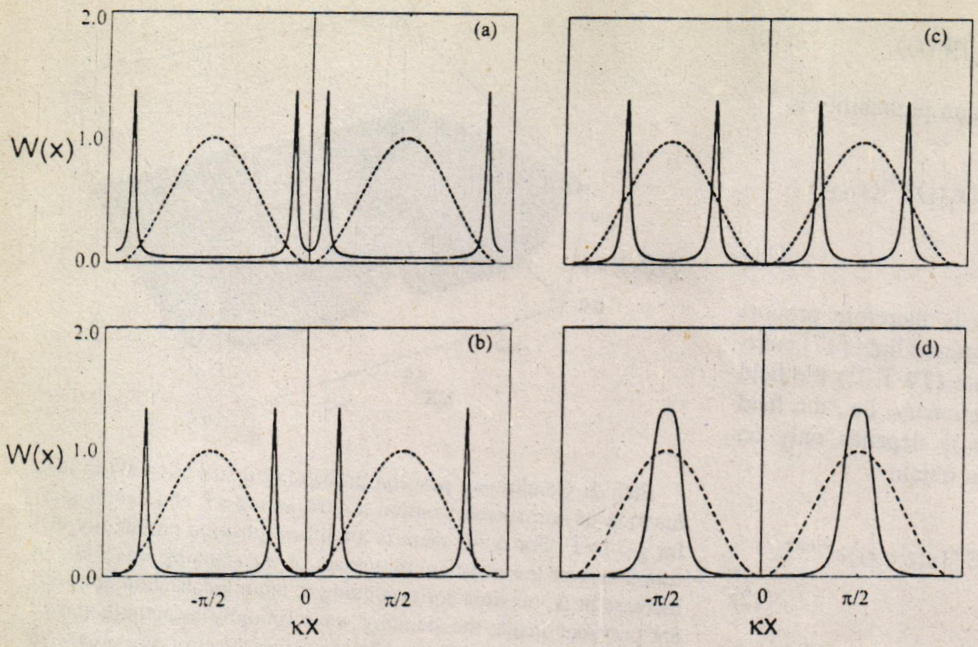

FIG. 3. Conditional position distribution $W(x)$ with $G/\Gamma=10$ for (a) $\Delta/\Gamma=5$, (b) $\Delta/\Gamma=10$, (c) $\Delta/\Gamma=15$, and (d) $\Delta/\Gamma=20$, clearly shows the dependence of position information on the detuning Δ. The solid line corresponds to the conditional position distribution $W(x)$ and the dotted line corresponds to the standing wave. Hence position information is available in the subwavelength domain of the standing-light field

and lie on the antinodes of the standing wave. There are noresonances for $|\Delta|>2G$ and we obtain a flat position distribution over the wavelength domain.

These results indicate a strong correlation between the detuning of the scattered light and the position of the atom. The measurement of a particular frequency corresponds to the localization of the atom in a subwavelength domain of the standing wave.

A clearer picture of the dependence of the localization scheme of an atom on the position-dependent Rabi frequency and detuning is demonstrated in Figs. 3(a)–3(d), where we show two-dimensional plots of the conditional position distribution $W(x)$ as a function of normalized position κx (ranging from $-\pi\rightarrow\pi$) for four different values of detuning, i.e., $\Delta/\Gamma=5,10,15,20$. The amplitude of the position-dependent Rabi frequency is taken to be $G/\Gamma=10$. It is clear from these plots that the best resolved peak is obtained at $\Delta=G$ for which the signal-to-background ratio is maximum. We get a partial overlap of the adjacent peaks for the ranges $0<|\Delta|<G$ and $G<|\Delta|<2G$. This causes an enhancement of the background. The strength of these overlaps and, consequently, the signal-to-background ratio depends on how much the detuning deviates from the maximum value of G. However, a complete overlap is observed for $\Delta=0$ and $\pm 2G$, which corresponds to the node and antinode, respectively.

We also investigate the dependence of the width of the best resolved peaks, for which the signal-to-background ratio is maximum, on the amplitude of the position-dependent Rabi frequency $g(x)=G\sin(\kappa x)$ (Fig. 4). It is noted that the width decreases with the increase in the amplitude G of the position-dependent Rabi frequency. The figure shows that the decrease in the width of the peak is very sharp for the values of G/Γ ranging from $2\rightarrow 20$. Outside this limit the width decreases slowly to a certain minimum value and stays practically asymptotic for $G/\Gamma>100$. This happens because, in this regime, the amplitude of the Rabi frequency is very large and the effect of the spontaneous emission, i.e., the linewidth, is minimized.

The spatial resolution in our scheme depends on the ratio of G/Γ and we must get a better spatial resolution for high Rabi frequency G as compared to the decay rate Γ. Apart from the periodicity of the standing wave which results four peaks in a conditional position distribution within a unit wavelength, a spatial resolution of $\approx\lambda/60$ can be achieved for a ratio of $G/\Gamma=10$. This is a reasonable approximation for the ratio G/Γ because recent experiments in the optical region on the realization of single atoms in the cavity QED reported a ratio of G/Γ of 8 [19] and in a more recent work it is enhanced to approximately 20 [20].

Here we mention again that the above power spectrum gives the conditional position distribution, i.e., the position information is conditioned on the measurement of the frequency of the emitted light. The frequency ω of the spontaneously emitted photon is related to the detuning parameter Δ, as $\omega=\omega_{ab}-\Delta$, where $\Delta=\pm 2g(x)$. Hence the detection of the spontaneously emitted photon gives the immediate information about the position of the atom inside the opti-

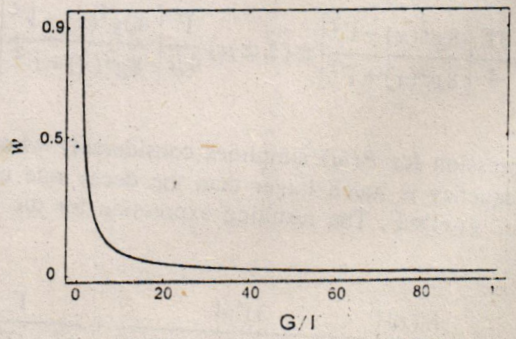

FIG. 4. Plot of width ($w\approx\kappa\Delta x$) versus G/Γ for the best resolved peaks ($\Delta=G$) in the conditional position distribution is The plot shows a strong dependence of w on the amplitude of position-dependent Rabi frequency G.

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calfield. Although the spontaneous-emission process is isotropic in nature and would require the use of 4π detectors in principle, for practical purposes it is not necessary to measure every atom.

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Amplitude and phase control of spontaneous emission

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We consider spontaneous emission in a four-level atomic system driven by three fields. It is shown that by controlling the phase and the amplitude of the driving fields a wide variety of spectral behavior can be obtained, ranging from a very narrow single spectral line to up to six spectral of varying widths.

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I. INTRODUCTION

Spontaneous emission in atomic systems arises due to the interaction of atoms with environmental modes. It is an interesting area of research to consider various means and systems to modify and control the spontaneous emission spectrum. We can control the fluorescence spectra by placing atoms in frequency-dependent reservoirs [1], in microwave cavities [2], or near the edges of photonic band gaps [3]. For atoms in free space, atomic coherence and quantum interference are the basic mechanisms for controlling the spontaneous emission. Control of spontaneous emission in atomic systems via quantum interference and atomic coherence results in a number of novel phenomena such as lasing without inversion [4], electromagnetically induced transparency [5], correlated spontaneous emission laser [6], absorption cancellation [7], and enhancement of the index of refraction with no absorption [8].

The quenching of spontaneous emission in an open V-type atom was studied in [9]. Phase-dependent effects in spontaneous emission spectra in a Λ-type atom were presented in Ref. [10] and for an atom near the edge of a photonic band gap in Refs. [11], [12]. Recently, Paspalakis and Knight proposed a phase control scheme in a four-level atom driven by two lasers of the same frequencies [13], where the relative phase of the two lasers was used to get partial cancellation, extreme linewidth narrowing, and total cancellation in the spontaneous emission spectrum. In these calculations, parallel dipoles for the two transitions were assumed. However, orthogonal dipoles for two transitions with small energy separation are easily found in nature. Therefore, it is worth considering the spectral linewidth narrowing and other effects for the case of two orthogonal dipoles, by controlling one phase and keeping the other constant.

In this paper we present another scheme for the four-level atom in which we can control the spontaneous emission by the amplitude and the phase of the driving fields. In our scheme, the quantum coherence is generated by a microwave field instead of the sharing of the vacuum modes by the two transitions. The proposed scheme requires three driving fields but is more convenient in its experimental realization.

We present analytical results for the spontaneous emission spectrum of a four-level atom. The upper two levels are closely spaced and are driven by the microwave field. These two levels are coupled with a third level via two coherent fields and decay to the fourth level. All the interactions are assumed to be resonant. We study the various effects of the dynamical variables, namely, the amplitudes or, more precisely, the Rabi frequencies and carrier phases, of the driven fields on the spontaneous emission spectrum. We predict six-peaks whose spectral behavior will be sensitive to these variables and for which their control will result in extreme partial cancellation and extreme linewidth narrowing. The linewidth narrowing is seen in the central peaks of the two sets of dressed states originating from slow decay rates. This is in agreement with the work of Zhou and Swain reported in [18], where they found linewidth narrowing in one of the dressed states near the quenching condition in the context of resonance fluorescence of a closed V-type atom. In this paper we assume that the transition frequency between the upper two levels is large as compared to their decay rates Γ_1 and Γ_2. This approximation allows us to neglect the quantum interference term proportional to $\sqrt{\Gamma_1\Gamma_2}$ in the equations of motion for the probability amplitudes. Our system is therefore independent of the alignment of the dipole moments. The trapping condition, however, is not physically achievable in this approximation.

The organization of this paper is as follows. In Sec. II we present the atomic model, the basic equations of motion, and their solution for the spontaneous emission spectrum. In Sec. III we analyze our results and discuss the dynamical variables that have the most direct influence on the shape of the spontaneous emission spectrum.

II. MODEL AND EQUATIONS

We consider a system of a four-level atom (see Fig. 1) interacting with three driving fields. These fields resonantly couple the transitions $|a_1\rangle$-$|b\rangle$, $|a_2\rangle$-$|b\rangle$, and $|a_1\rangle$-$|a_2\rangle$ with Rabi frequencies Ω_1, Ω_2, and Ω_3, respectively. The upper levels $|a_1\rangle$ and $|a_2\rangle$ decay to the lower level $|c\rangle$ via interactions with the vacuum field modes. The interaction picture Hamiltonian in the dipole and rotating-wave approximation is given by

$$H(t)=\hbar(\Omega_1|a_1\rangle\langle b|+\Omega_2|a_2\rangle\langle b|+\Omega_3|a_1\rangle\langle a_2|)$$
$$+\hbar\sum_k (g_k^{(1)}e^{i(\omega_{1c}-\nu_k)t}|a_1\rangle\langle c|b_k+g_k^{(2)}e^{i(\omega_{2c}-\nu_k)t}|a_2\rangle$$
$$\times\langle c|b_k)+\text{H.c.}, \quad (1)$$

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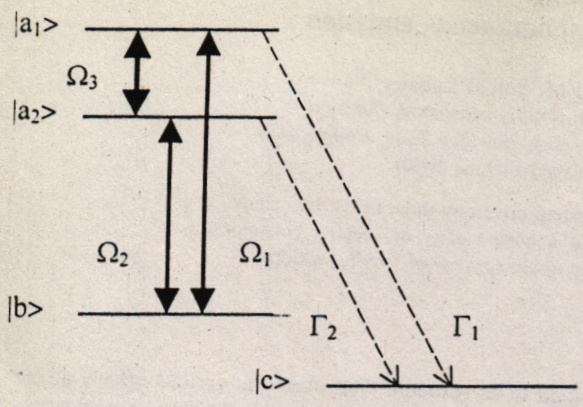

FIG. 1. Level scheme of the model atom. The bold double arrow indicates the microwave field coupled with upper levels $|a_1\rangle$ and $|a_2\rangle$. The same upper two levels are coupled with $|b\rangle$ via two coherent fields and with $|c\rangle$ via common field modes, shown by solid double arrows and dashed single arrows, respectively.

where $b_{\mathbf{k}}$ and $b_{\mathbf{k}}^{\dagger}$ are the annihilation and creation operators for the reservoir modes with wave vector $\mathbf{k}$ and frequency $\nu=c|\mathbf{k}|=ck$, and $g_{\mathbf{k}}^{(1,2)}$ are the coupling constants between the $\mathbf{k}$th mode and the atomic dipoles between levels $|a_1\rangle,|a_2\rangle$ and the level $|c\rangle$. We assume the coupling constants to be real for the sake of simplicity. Here ω_{1c},ω_{2c} are the transition frequencies from levels $|a_1\rangle,|a_2\rangle$ to $|c\rangle$, respectively. At any time t, the atom-field state vector can be written as

$$|\Psi(t)\rangle=[A_1(t)|a_1\rangle+A_2(t)|a_2\rangle+B(t)|b\rangle]|\{0\}\rangle + \sum_{\mathbf{k}} C_{\mathbf{k}}(t)|c\rangle|1_{\mathbf{k}}\rangle, \tag{2}$$

where $|\{0\}\rangle$ denotes the vacuum of the electromagnetic field. Using Weisskopf-Wigner theory [14], the equations of motion for the probability amplitudes are given by

$$\dot{B}(t)=-i\Omega_1^* A_1(t)-i\Omega_2^* A_2(t), \tag{3}$$

$$\dot{A}_1(t)=-i\Omega_1 B(t)-\frac{\Gamma_1}{2}A_1(t)-i\Omega_3 A_2(t) - p\frac{\sqrt{\Gamma_1\Gamma_2}}{2}A_2(t)e^{i\omega_{12}t}, \tag{4}$$

$$\dot{A}_2(t)=-i\Omega_2 B(t)-i\Omega_3^* A_1(t)-\frac{\Gamma_2}{2}A_2(t) - p\frac{\sqrt{\Gamma_1\Gamma_2}}{2}A_2(t)e^{-i\omega_{12}t}, \tag{5}$$

$$\dot{C}_{\mathbf{k}}(t)=-iA_1(t)g_{\mathbf{k}}^{(1)}e^{i(\nu_{\mathbf{k}}-\omega_{1c})t}-iA_2(t)g_{\mathbf{k}}^{(2)}e^{i(\nu_{\mathbf{k}}-\omega_{2c})t}, \tag{6}$$

where $\Gamma_j(j=1,2)$ are the radiative decay rates from the upper two levels to the lower level, respectively, and p denotes the alignment of the matrix elements of the two dipole moments and is given by

$$p=\frac{\langle a_1|\mathbf{r}|c\rangle\cdot\langle a_2|\mathbf{r}|c\rangle}{|\langle a_1|\mathbf{r}|c\rangle||\langle a_2|\mathbf{r}|c\rangle|}. \tag{7}$$

If the matrix elements are orthogonal, there is no interference between the decay paths $|a_1\rangle$-$|c\rangle$ and $|a_2\rangle$-$|c\rangle$ and $p=0$, and if the matrix elements are parallel, there is maximum interference and $p=1$. We neglect the last terms in Eqs. (4) and (5), however, under the approximation $\omega_{12}\gg\Gamma_{1,2}$ [15].

Next we solve Eqs. (3)–(5) for the steady-state expression for the probability amplitude $C_{\mathbf{k}}(t\to\infty)$. On integrating Eq. (6) we obtain

$$C_{\mathbf{k}}(t\to\infty)=-ig_{\mathbf{k}}^{(1)}\mathcal{A}_1(s=-i\delta_1)-ig_{\mathbf{k}}^{(2)}\mathcal{A}_2(s=-i\delta_2), \tag{8}$$

where $\mathcal{A}_i(s)$ are the Laplace transforms of the probability amplitudes $A_i(t)(i=1,2)$, i.e.,

$$\mathcal{A}_i(s)=\int_0^{\infty}A_i(t)e^{-st}dt, \tag{9}$$

and $\delta_1=\nu_{\mathbf{k}}-\omega_{1c}$ and $\delta_2=\nu_{\mathbf{k}}-\omega_{2c}$. In the following we use $\delta=\nu_{\mathbf{k}}-\omega_{1c}+\omega_{12}/2$. We then have $\delta_1=\delta-\omega_{12}/2$ and $\delta_2=\delta+\omega_{12}/2$. Using the approximation discussed above, taking the Laplace transform of Eqs. (3)–(5), and using Cramer's rule to evaluate the transformed $\mathcal{A}_1(s),\mathcal{A}_2(s)$, we get the following expression for $C_{\mathbf{k}}(t\to\infty)$:

$$C_{\mathbf{k}}(t\to\infty)=g_{\mathbf{k}}^{(1)}\frac{M_1(\delta)}{N_1(\delta)}+g_{\mathbf{k}}^{(2)}\frac{M_2(\delta)}{N_2(\delta)}, \tag{10}$$

where

$$M_1(\delta)=B(0)\left[\Omega_1\left(\delta-\frac{\omega_{12}}{2}+i\frac{\Gamma_1}{2}\right)+\Omega_2\Omega_3\right] +A_1(0)\left[\left(\delta-\frac{\omega_{12}}{2}\right)\left(\delta-\frac{\omega_{12}}{2}+i\frac{\Gamma_2}{2}\right)-|\Omega_2|^2\right] +A_2(0)\left[\left(\delta-\frac{\omega_{12}}{2}\right)\Omega_3+\Omega_1^*\Omega_2\right], \tag{11}$$

$$M_2(\delta)=B(0)\left[\Omega_1\Omega_3^*+\Omega_2\left(\delta+\frac{\omega_{12}}{2}+i\frac{\Gamma_2}{2}\right)\right]+A_1(0) \times\left[\left(\delta+\frac{\omega_{12}}{2}\right)\Omega_3^*+\Omega_1^*\omega_2\right]+A_2(0)\left[\left(\delta+\frac{\omega_{12}}{2}\right) \times\left(\delta+\frac{\omega_{12}}{2}+i\frac{\Gamma_1}{2}\right)-|\Omega_1|^2\right], \tag{12}$$

$$N_1(\delta)=\left(\delta-\frac{\omega_{12}}{2}\right)\left(\delta-\frac{\omega_{12}}{2}+i\frac{\Gamma_1}{2}\right)\left(\delta-\frac{\omega_{12}}{2}+i\frac{\Gamma_2}{2}\right)$$
$$-|\Omega_1|^2\left(\delta-\frac{\omega_{12}}{2}+i\frac{\Gamma_2}{2}\right)-|\Omega_2|^2\left(\delta-\frac{\omega_{12}}{2}+i\frac{\Gamma_1}{2}\right)$$
$$-\left(\delta-\frac{\omega_{12}}{2}\right)|\Omega_3|^2-(\Omega_1^*\Omega_2\Omega_3+\Omega_1\Omega_2^*\Omega_3^*), \quad (13)$$

$$N_2(\delta)=\left(\delta+\frac{\omega_{12}}{2}\right)\left(\delta+\frac{\omega_{12}}{2}+i\frac{\Gamma_1}{2}\right)\left(\delta+\frac{\omega_{12}}{2}+i\frac{\Gamma_2}{2}\right)$$
$$-|\Omega_1|^2\left(\delta+\frac{\omega_{12}}{2}+i\frac{\Gamma_2}{2}\right)-|\Omega_2|^2\left(\delta+\frac{\omega_{12}}{2}+i\frac{\Gamma_1}{2}\right)$$
$$-\left(\delta+\frac{\omega_{12}}{2}\right)|\Omega_3|^2-(\Omega_1^*\Omega_2\Omega_3+\Omega_1\Omega_2^*\Omega_3^*). \quad (14)$$

To analyze the spontaneous emission spectrum we assume the atom to be initially prepared in the state $|b\rangle$ so that $B(0)=1$ and $A_1(0)=A_2(0)=0$. The coupling constants $g_k^{(1)}$ and $g_k^{(2)}$ are chosen such that $g_k^{(1)}=g_k^{(2)}=1$. Further, to account for the effect of the phase of the microwave field on the spontaneous emission spectrum, we replace Ω_3 by $\Omega_3|e^{i\varphi}$. We assume Ω_1 and Ω_2 to be real, i.e., $\Omega_1=|\Omega_1|$ and $\Omega_2=|\Omega_2|$. We can then write a general expression of the probability amplitude for any values of spectroscopic parameters as

$$C_k(t\to\infty)=\frac{\alpha_1+i\beta_1}{\delta-(\gamma_1+i\zeta_1)}+\frac{\alpha_2+i\beta_2}{\delta-(\gamma_2+i\zeta_2)}+\frac{\alpha_3+i\beta_3}{\delta-(\gamma_3+i\zeta_3)}$$
$$+\frac{\alpha_4+i\beta_4}{\delta-(\gamma_4+i\zeta_4)}+\frac{\alpha_5+i\beta_5}{\delta-(\gamma_5+i\zeta_5)}$$
$$+\frac{\alpha_6+i\beta_6}{\delta-(\gamma_6+i\zeta_6)}. \quad (15)$$

Here the quantities α_i, β_i, γ_i, and ζ_i $(i=1-6)$ depend on the spectroscopic parameters, chosen such that γ_i and ζ_i are the real and imaginary parts of λ_i for $i=1-3$ and μ_i for $i=4-6$ that satisfy the cubic equations $N_1(\lambda)=0$ and $N_2(\mu)=0$. α_i and β_i are the real and imaginary parts of $X_i(i=1-6)$ that are given by

$$X_1=\frac{(\lambda_2-\lambda_3)}{D_1}\left(i|\Omega_1|\frac{\Gamma_1}{2}+|\Omega_2||\Omega_3|e^{i\varphi}-|\Omega_1|\frac{\omega_{12}}{2}\right.$$
$$\left.+|\Omega_1|\lambda_1\right),$$

$$X_2=\frac{(\lambda_3-\lambda_1)}{D_1}\left(i|\Omega_1|\frac{\Gamma_1}{2}+|\Omega_2||\Omega_3|e^{i\varphi}-|\Omega_1|\frac{\omega_{12}}{2}\right.$$
$$\left.+|\Omega_1|\lambda_2\right),$$

$$X_3=\frac{(\lambda_1-\lambda_2)}{D_1}\left(i|\Omega_1|\frac{\Gamma_1}{2}+|\Omega_2||\Omega_3|e^{i\varphi}-|\Omega_1|\frac{\omega_{12}}{2}\right.$$
$$\left.+|\Omega_1|\lambda_3\right),$$

$$X_4=\frac{(\mu_5-\mu_6)}{D_2}\left(i|\Omega_2|\frac{\Gamma_2}{2}+|\Omega_1||\Omega_3|e^{-i\varphi}+|\Omega_2|\frac{\omega_{12}}{2}\right.$$
$$\left.+|\Omega_2|\mu_4\right),$$

$$X_5=\frac{(\mu_6-\mu_4)}{D_2}\left(i|\Omega_2|\frac{\Gamma_2}{2}+|\Omega_1||\Omega_3|e^{-i\varphi}+|\Omega_2|\frac{\omega_{12}}{2}\right.$$
$$\left.+|\Omega_2|\mu_5\right),$$

$$X_6=\frac{(\mu_4-\mu_5)}{D_2}\left(i|\Omega_2|\frac{\Gamma_2}{2}+|\Omega_1||\Omega_3|e^{-i\varphi}+|\Omega_2|\frac{\omega_{12}}{2}\right.$$
$$\left.+|\Omega_2|\mu_6\right),$$

with

$$D_1=\lambda_1^2(\lambda_2-\lambda_3)+\lambda_2^2(\lambda_3-\lambda_1)+\lambda_3^2(\lambda_1-\lambda_2),$$
$$D_2=\mu_4^2(\mu_5-\mu_6)+\mu_5^2(\mu_6-\mu_4)+\mu_6^2(\mu_4-\mu_5).$$

The spontaneous emission spectrum $S(\delta)$ is proportional to $|C_k(t\to\infty)|^2$. Thus, apart from a proportionality constant, the spontaneous emission spectrum is given by

$$S(\delta)=\Gamma_1\left|\frac{\alpha_1+i\beta_1}{\delta-(\gamma_1+i\zeta_1)}+\frac{\alpha_2+i\beta_2}{\delta-(\gamma_2+i\zeta_2)}+\frac{\alpha_3+i\beta_3}{\delta-(\gamma_3+i\zeta_3)}\right|^2$$
$$+\Gamma_2\left|\frac{\alpha_4+i\beta_4}{\delta-(\gamma_4+i\zeta_4)}+\frac{\alpha_5+i\beta_5}{\delta-(\gamma_5+i\zeta_5)}\right.$$
$$\left.+\frac{\alpha_6+i\beta_6}{\delta-(\gamma_6+i\zeta_6)}\right|^2. \quad (16)$$

The spontaneous emission spectrum given in Eq. (16) consists of two parts. Each part corresponds to three peaks associated with the three dressed states of which it is composed. In Eq. (16), we neglected the interference terms between the two sets of dressed states corresponding to the two bare states due to the large separation between them. The spectrum therefore consists in general of six peaks located at $\delta=\gamma_i$ $(i=1-6)$. In many situations of interest, the interference terms occurring in the spectrum equation have negligible contributions; thus the heights of the peaks located at $\delta=\gamma_i$ are given by $(\alpha_i^2+\beta_i^2)/\zeta_i^2$ for $i=1-6$.

We examine the condition for trapping in this system. In order to have a nonvanishing steady-state population in the upper states of the system, the constant term of its characteristic equation is set to zero. The resulting condition for population trapping is

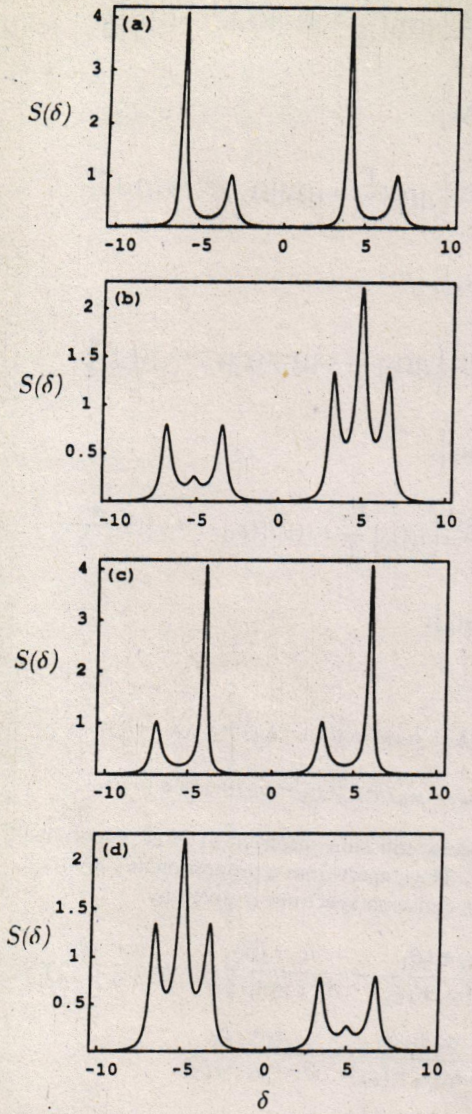

FIG. 2. The spontaneous emission spectra $S(\delta)$ (in units of Γ^{-1}) for $\Omega_1, \Omega_2, |\Omega_3| = \Gamma$. $\varphi =$ (a) 0, (b) $\pi/2$, (c) π, (d) $3\pi/2$.

$$2|\Omega_1||\Omega_2||\Omega_3|\cos(\varphi) + i\left(|\Omega_1|^2\frac{\Gamma_2}{2} + |\Omega_2|^2\frac{\Gamma_1}{2}\right) = 0. \tag{17}$$

In the last equation, the real part can be zero if $\varphi = \pi/2$, while the vanishing of the imaginary part requires negative decay rates, which is not physically allowed. There is therefore no trapping state in our system. It may be pointed out that the terms $p\sqrt{\Gamma_1\Gamma_2}e^{\mp i\omega_{12}t}$ occurring in Eqs. (4) and (5) are the sources of quantum interference [17] and contribute to the trapping conditions. However, in our analysis they are neglected under the approximation $\omega_{12} \gg \Gamma_{1,2}$. Thus the behavior is different from that reported in [13], where the dependence on the alignment of the the dipole moments led to trapping conditions. No such conditions exist in our system.

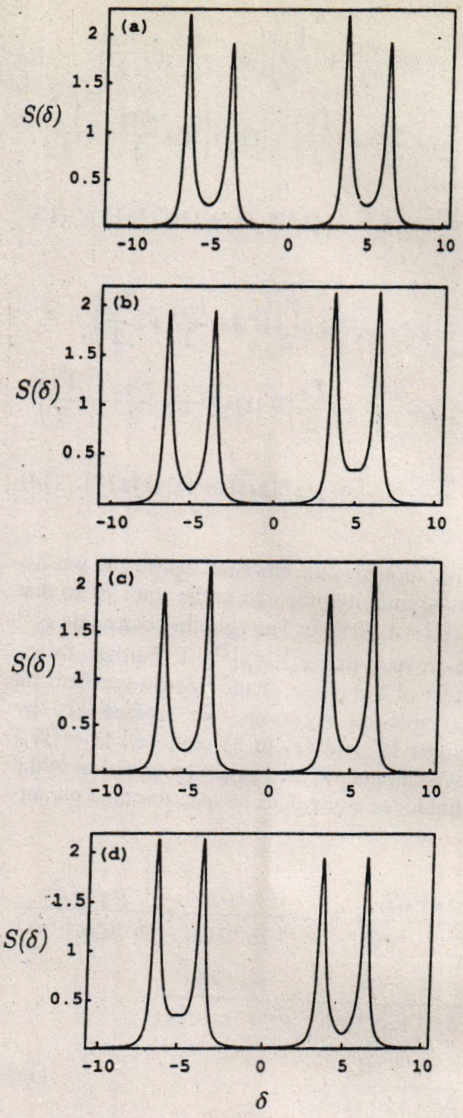

FIG. 3. The spontaneous emission spectra $S(\delta)$ (in units of Γ^{-1}) for $\Omega_1, \Omega_2 = \Gamma$ and $|\Omega_3| = 0.1\Gamma$. $\varphi =$ (a) 0, (b) $\pi/2$, (c) π, (d) $3\pi/2$.

III. RESULTS AND DISCUSSION

Our system reduces to the usual form of Autler-Townes scheme where the spontaneous emission spectrum is split into doublets [16] when the atom is initially prepared in the state $|a_2\rangle$ and the Rabi frequencies Ω_1, Ω_2 as well as the decay rate Γ_1 are equal to zero. If the atom is initially prepared in a coherent superposition of the upper two levels $|\Psi(0)\rangle = (e^{i\varphi_p}|a_1,\{0\}\rangle + |a_2,\{0\}\rangle)/\sqrt{2}$, and the decay rates Γ_1 and Γ_2 are nonzero, there are four peaks in the spectrum originating from dynamical Stark splitting of the upper two levels [19]. The variation of the relative phase of the pump and driving lasers results in a similar effect to the one reported recently [17].

Next we discuss the spectrum as given by Eq. (16). The equation contains two major parts due to the two bare upper

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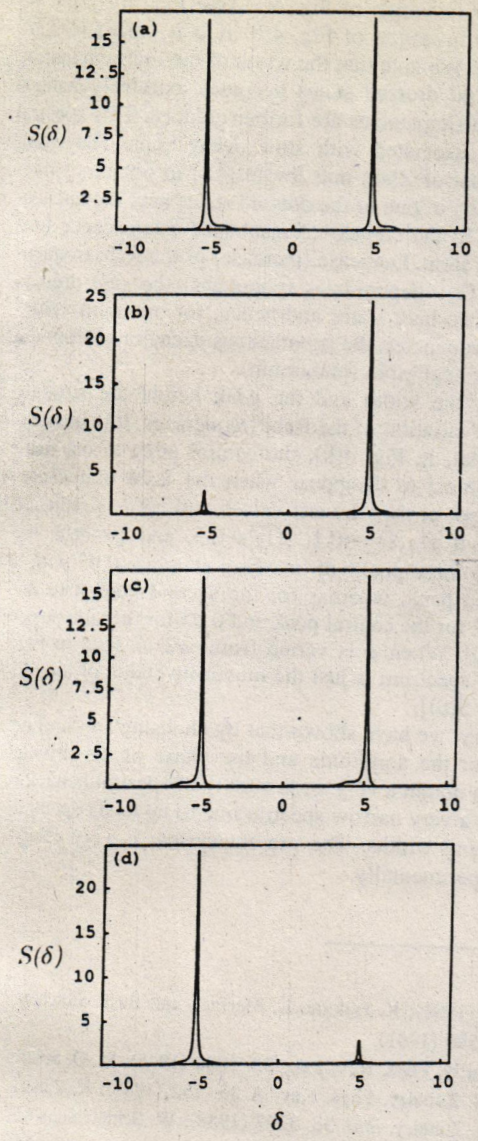

FIG. 4. The spontaneous emission spectra $S(\delta)$ (in units of Γ^{-1}) for $\Omega_1, \Omega_2 = 0.3\Gamma$ and $|\Omega_3| = \Gamma$. $\varphi =$ (a) 0, (b) $\pi/2$, (c) π, (d) $3\pi/2$.

states. Each part contains three terms corresponding to three peaks associated with the three dressed states. The equation therefore leads to a spectral behavior consisting of six peaks. The interference terms occurring in the equation have negligible contributions. Therefore, the peak heights contributed by the two bare states are $(\alpha_i^2+\beta_i^2)/\zeta_i^2 (i=1-3)$ and $(\alpha_i^2+\beta_i^2)/\zeta_i^2$ $(i=4-6)$, respectively. In what follows, we assume $g_k^{(1)} = g_k^{(2)}$, $\Gamma_1 = \Gamma_2 = \Gamma$, and $\omega_{12} = 10\Gamma$. In the following discussion the Rabi frequencies Ω_1, Ω_2, and Ω_3 are given in units of Γ.

We consider the effects of the dynamical variables, namely, the amplitudes, or more precisely the Rabi frequencies, and carrier phases, of the driven fields on the spontaneous emission spectrum. The variation of the phase φ associated with the microwave field influences the spontaneous emission spectrum efficiently. In the spectrum equation all the terms, except the central terms, are significant when $\Omega_1, \Omega_2, |\Omega_3| = 1$ and $\varphi = 0$. The plot for these values shows an extremely suppressed central peak and enhanced side peaks. Furthermore, for the two bare states, the height of one is larger than the other [see Fig. 2(a)]. The peak heights are 4.2, 0.025, and 1 for the two bare states. The central terms reach their maximum when φ is varied from 0 to $\pi/2$ in Fig. 2(a). Now all terms are significant. In this case, the plot shows a suppressed central peak and equally enhanced side peaks for the one bare state and vice versa for the second [see Fig. 2(b)]. Here the peak heights are 0.80, 0.28, 0.80 and 1.35, 2.25, 1.35 for the two bare states. When φ is further varied from $\pi/2$ to π in Fig. 2(b), the new spectrum is just the mirror inversion of Fig. 2(a) [see Fig. 2(c)], and we get the mirror inversion of Fig. 2(b) if φ is varied to $3\pi/2$ [see Fig. 2(d)]. We note that the peak height varies with φ; however, there is no appreciable change in the position of the spectral lines on the frequency axes. This behavior is in agreement with the coherently driven three-level Λ-type atom of Martinez *et al.* [10]. Moreover, the occurrence of a mirror image at $\varphi + \pi$ also agrees with the said reference. The enhancement around $\pi/2$, $3\pi/2$ and strong suppression around $0, \pi$ of the central peaks is in accord with the work of Paspalakis and Knight reported recently, where they used the relative phase of two lasers of the same frequencies to control the three-peak spontaneous emission spectrum in a four-level atom [13].

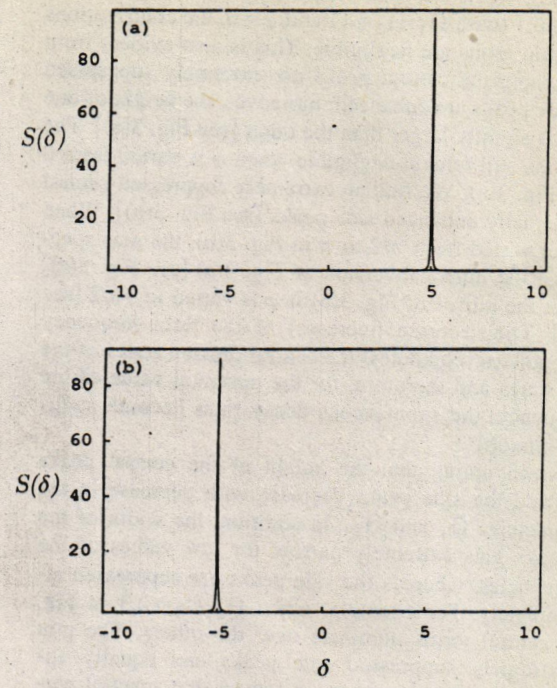

FIG. 5. The spontaneous emission spectra $S(\delta)$ (in units of Γ^{-1}) for $\Omega_1, \Omega_2 = 0.1\Gamma$ and $|\Omega_3| = 0.5\Gamma$. $\varphi =$ (a) $\pi/2$, (b) $3\pi/2$.

The shape of the spontaneous emission spectrum is strongly influenced by the variation of the Rabi frequencies.

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For instance, when the Rabi frequency $|\Omega_3|$ in Fig. 2(a) is reduced to 0.1 (with $\Omega_1,\Omega_2=1$) and $\varphi=0$, the contributions of the central terms are negligible. This is also evident from the plot, where the central peaks are extremely suppressed and the side peaks are enhanced; moreover, the height of one side peak is slightly larger than the other [see Fig. 3(a)]. The central terms still remain negligible when φ is varied from 0 to $\pi/2$ in Fig. 3(a). We find an extremely suppressed central peak but equally enhanced side peaks [see Fig. 3(b)]. When φ is further varied from $\pi/2$ to π in Fig. 3(b), the new spectrum is just the mirror inversion of Fig. 3(a) [see Fig. 3(c)] and we get the mirror of Fig. 3(b) if φ is varied to $3\pi/2$ [see Fig. 3(d)]. The decrease (increase) of the Rabi frequency $|\Omega_3|$ depopulates (populates) the central dressed states of the two bare states and therefore, for the optimum value of the Rabi frequency, the spontaneous decay rates become negligible (maximum).

It is worth noting that the height of the central peaks increases and the side peaks decrease with decrease in the Rabi frequencies Ω_1 and Ω_2. In addition, the width of the central peaks gets extremely narrow for low values of the Rabi frequencies, whereas the side peaks are suppressed almost completely. For example, when $\Omega_1,\Omega_2=0.3$ in Fig. 2(a), the central terms dominate over the others. The plot shows extremely suppressed side peaks and equally enhanced central peaks. Moreover, a remarkable spectral narrowing is also seen [see Fig. 4(a)]. The peak heights in this case are 9×10^{-33}, 17, 0.6 for the two bare states. On varying φ from 0 to $\pi/2$, one of the central terms increases while the other decreases. This is also clear from the plot, where the central peaks are enhanced and one peak is larger than the other [see Fig. 4(b)]. In this case the peak heights are 0.03, 2.8, 0.03 and 1.5, 24.6, 1.5 for the two bare states. When φ is further varied from $\pi/2$ to π, the new spectrum is just the mirror inversion of Fig. 4(a) [see Fig. 4(c)] and we get the mirror inversion of Fig. 4(b) if φ is varied to $3\pi/2$ [see Fig. 4(d)]. We note that the width of the lines emanating from the central dressed states becomes extremely narrow when the Rabi frequencies are further reduced. This spectral narrowing is associated with slow decay rates. The result agrees with that of Zhou and Swain [18] in obtaining linewidth narrowing of one of the dressed states near the quenching condition in the context of resonance fluorescence of a closed V-type atom. Decrease.(increase) of the Rabi frequencies Ω_1 and Ω_2, depopulates (populates) the side dressed states of the two bare states and hence, for optimum values of the Rabi frequencies, the spontaneous decay rates from the states become negligible (maximum).

Obviously, the width and the peak height are strongly influenced by variation of the Rabi frequencies. It is interesting to note that, in Fig. 4(b), the central peak of one bare state starts almost to disappear when the Rabi frequencies Ω_1,Ω_2 are kept at their lowest values and $|\Omega_3|$ is reduced. For instance, if $\Omega_1,\Omega_2=0.1$, $|\Omega_3|=0.5$, and $\varphi=\pi/2$, the heights of the three peaks of one bare state are 0.05 with a narrow central peak, whereas for the second bare state the heights are 87 for the central peak and 0.05 for the side peaks [see Fig. 5(a)]. When φ is varied from $\pi/2$ to $3\pi/2$ in Fig. 5(a), the new spectrum is just the mirror inversion of the old one [see Fig. 5(b)].

In summary, we have shown that by choosing appropriate parameters for the amplitude and the phase of the driving fields, we can obtain a very wide variety of spectral behavior ranging from a very narrow spectral line to up to six spectral lines of varying widths. The present system is very easily realizable experimentally.

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Quantum-state tomography using phase-sensitive amplification

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We propose a model for the quantum-state measurement via phase-sensitive amplification. The basic idea is to amplify the quantum state through a two-photon correlated-emission laser, such that there is no noise in the quadrature of interest and all the noise is fed into the conjugate quadrature. The noise-free quadrature is prepared in different phases and then corresponding quadrature distribution is measured. The Wigner function of the initial quantum state is then reconstructed by carrying out inverse Radon transformation familiar in tomographic imaging. This scheme allows us to avoid the deterioration of homodyne detection measurement due to the problem of detector efficiency.

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I. INTRODUCTION

Quantum-state measurement has been a subject of great interest in recent years [1–4]. As all the information of a quantum system is contained in the density matrix ρ of the system, so the measurement of the density matrix elements will completely characterize the given quantum state. The Wigner function of a quantum state bears a one-to-one correspondence with the density matrix ρ of the state [5]. Once the Wigner function of a quantum state is known then the corresponding density matrix elements of the state can be worked out by employing the Wigner formula [6,7]. On the measurement side, a balanced homodyne detector measures the linear combination of the creation and the annihilation operators $\{\frac{1}{2}[a^{\dagger}\exp(i\theta)+a\exp(-i\theta)]\}$ of a quantized field [8,9]. This linear combination of creation and annihilation operators is also termed as the generalized or rotated quadrature of the field and the phase θ of this quadrature is given by the phase of the local oscillator in the balanced homodyne detection scheme. Two specific phases, $\theta=0$ and $\theta=\pi/2$, of this generalized quadrature $x(\theta)$ are the same quadrature phases $x(0)$ and $x(\pi/2)$ as have been introduced in relation to squeezed and coherent states of a field [6,10].

Vogel and Risken [11] have shown that the quasiprobability distributions such as P, Q, and the Wigner function bear a one-to-one correspondence with the generalized quadrature distribution function $\omega(x,\theta)$. From a set of measurements of the generalized quadrature amplitude $x(\theta)$ in the balanced homodyne detection scheme, the quadrature distribution $\omega(x,\theta)$ can be known, and hence by tomographic imaging of this distribution, the P, Q, and the Wigner function can be obtained. Following the same scheme, Faridani *et al.* [12] and later Mlynek *et al.* [13] have experimentally measured the quantum state of the radiation field. Recently, some other methods have also been proposed for the measurement of the quantum state of the radiation field. These include methods based on absorption and emission spectroscopy [14], the conditional measurements on the atoms in a micromaser [15], dispersive atom-field coupling in Ramsey method of separated oscillatory fields [16], as well as some others [17].

However, the quantum objects are highly feeble and delicate entities. Their subtleties remain highly obscured in the measurement process owing much to the detectors inefficiencies. In some recent studies, it is shown that the measured quadrature distribution $\omega(x,\theta)$ becomes smoothed due to the finite detection efficiency [18,19]. As a result, instead of the Wigner function, smoothed quasiprobabilities are constructed [19]. In this paper, we propose a scheme for the measurement of quantum state of the radiation field using two-photon correlated-emission laser (CEL) [20–25]. During the amplification through a phase-sensitive amplifier, there is no noise in the quadrature of interest and all the noise is fed into the conjugate quadrature. Therefore, the quantum information remains intact in one quadrature phase of the field and may be extracted out of it for the construction of quantum state of the field.

In order to construct the Wigner function of the quantum state, we require a set of distribution functions $\omega(x,\theta)$ for quadrature values $x(\theta)$ for θ varying from 0 to π. To obtain noise-free amplification for different quadrature phases, we prepare the amplifier in different phases φ, accordingly. We have calculated the quadrature distributions for any arbitrary quantum state after its amplification through a phase-sensitive amplifier. The distribution function of the noise-free quadrature is then used to construct the Wigner function of the quantum state using quantum tomography. We apply this model to a Schrödinger cat state [26] and discuss the reconstruction of the corresponding Wigner function after its amplification through a two-photon CEL. Our proposed method is insensitive to detector efficiency which poses serious problems in observing the nonclassical features associated with the quantum state. In a recent paper, we have shown that the quantum interferences associated with a Schrödinger cat state can be observed using phase-sensitive linear amplification [27]. It may be pointed out that the phase-sensitive amplification of the Schrödinger cat state and the resulting nonclassical characteristics during the amplification process are discussed in Refs. [25,28].

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II. MEASUREMENT OF THE QUANTUM STATE USING TWO-PHOTON CEL

We consider a two-photon phase sensitive linear amplifier [20], which consists of three-level atoms in cascade configuration. The atoms are initially prepared in a coherent superposition of levels $|a\rangle$ and $|c\rangle$, i.e., the initial density operator for the atoms is given by

$$\rho_i=\rho_{aa}|a\rangle\langle a|+\rho_{ac}|a\rangle\langle c|+\rho_{ca}|c\rangle\langle a|+\rho_{cc}|c\rangle\langle c|. \quad (1)$$

We assume that such atoms are injected at a random injection rate R inside the cavity where they interact with the field for a time τ (see Fig. 1). It is assumed that the cavity field is resonant with the atomic transitions $|a\rangle-|b\rangle$ and $|b\rangle-|c\rangle$ and $R\tau<1$ such that there is only one atom at a time inside the cavity. The evolution of the reduced density matrix of the field ρ_F is given by the following master equation [6]:

$$\begin{aligned}\dot{\rho}_F=&-\frac{A}{2}(N+1)[aa^\dagger\rho_F-2a^\dagger\rho_F a+\rho_F aa^\dagger]\\&-\frac{A}{2}N[a^\dagger a\rho_F-2a\rho_F a^\dagger+\rho_F a^\dagger a]\\&-\frac{A}{2}M^*[aa\rho_F-2a\rho_F a+\rho_F aa]\\&-\frac{A}{2}M[a^\dagger a^\dagger\rho_F-2a^\dagger\rho_F a^\dagger+\rho_F a^\dagger a^\dagger],\end{aligned} \quad (2)$$

where $A=Rg^2\tau^2(\rho_{aa}-\rho_{cc})$ is the gain coefficient. Here g is the atom-field interaction constant and ρ_{aa} and ρ_{cc} are the density matrix elements corresponding to atom in level a and c, respectively. The constants N and M are defined as

$$N=\frac{\rho_{cc}}{(\rho_{aa}-\rho_{cc})},\qquad M=\frac{\rho_{ac}}{(\rho_{aa}-\rho_{cc})}. \quad (3)$$

the terms proportional to M contain the phase-sensitivity of the coherent atomic superposition.

The Wigner function $W(\alpha,t)$ is defined in terms of the density operator ρ_F by [6]

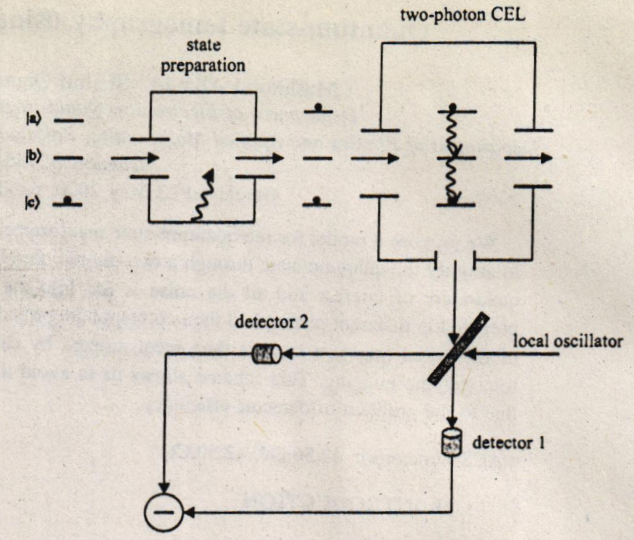

FIG. 1. The schematic diagram of the two-photon phase-sensitive linear amplifier and the measurement of the noise free quadrature via balanced homodyne detection scheme.

$$W(\alpha,t)=\pi^{-2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}d^2\beta\ \mathrm{Tr}\{\exp[-\beta(\alpha^*-a^\dagger)+\beta^*(\alpha-a)]\rho_F\}. \quad (4)$$

The master equation (2) for the reduced density matrix ρ_F can be rewritten as the following Fokker-Planck equation for the Wigner function:

$$\frac{\partial}{\partial t}W=-\frac{A}{2}\left(\frac{\partial}{\partial t}\alpha+\frac{\partial}{\partial t}\alpha^*+M\frac{\partial^2}{\partial\alpha^2}+M^*\frac{\partial^2}{\partial\alpha^{*2}}-2(N+1/2)\frac{\partial^2}{\partial\alpha\partial\alpha^*}\right)W. \quad (5)$$

A solution of this equation yields the evolution of the Wigner function for any arbitrary initial quantum state [29]

$$W(\alpha,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}d^2\beta W(\beta,0)W_c(\alpha,t;\beta,0), \quad (6)$$

where the conditional probability $W_c(\alpha,t;\beta,0)$ reads as

$$W_c(\alpha,t;\beta,0)=\frac{1}{\pi(G-1)\sqrt{[(N+1/2)^2-|M|^2]}}\times\exp\left(-\frac{(|\alpha|\cos(\vartheta-\varphi/2)-\sqrt{G}|\beta|\cos(\theta_0-\varphi/2)^2}{[N+1/2-|M|](G-1)}-\frac{[|\alpha|\sin(\vartheta-\varphi/2)-\sqrt{G}|\beta|\sin(\theta_0-\varphi/2)]^2}{[N+1/2+|M|](G-1)}\right). \quad (7)$$

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Here $G=\exp(At)$ is defined as the gain factor and the complex quantities α, β, and ρ_{ac} are expressed in the polar forms as, $\alpha=|\alpha|\exp(i\vartheta)$, $\beta=|\beta|\exp(i\theta_0)$, and $\rho_{ac}=|\rho_{ac}|\exp(i\varphi)$. In the case of perfect coherence we have the relation $|\rho_{ac}|=\sqrt{\rho_{aa}\rho_{cc}}$. We therefore, define a squeezing parameter r such that [30]

$$\tanh^2 r=\frac{\rho_{cc}}{\rho_{aa}}. \tag{8}$$

In terms of the squeezing parameter r, the constants N and $|M|$ are defined as

$$N=\sinh^2(r),$$

$$|M|=\frac{\sinh(2r)}{2}. \tag{9}$$

Here we look at the measurement of the quadrature distribution $\omega(x,\theta)$ for the amplified quantum state. A homodyne detector measures the quadrature component

$$x(\theta)=x(\theta)^{\dagger}=\frac{(a^{\dagger}e^{i\theta}+a\,e^{-i\theta})}{2}. \tag{10}$$

In a balanced homodyne experiment, θ can be varied by shifting the local oscillator phase. A complete distribution for $x(\theta)$ is given by the quadrature distribution $\omega(x,\theta)$. Such distributions have recently been measured employing quantum optical tomography.

The quadrature distribution $\omega(x,\theta)$ for the amplified field can be obtained from the Wigner function $W(\alpha,t)$ by using the following relation [11]:

$$\omega(x,\theta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\alpha\,d\eta\,W(\alpha,t)\exp[-i\eta(x-\alpha_x\cos\theta-\alpha_y\sin\theta)]. \tag{11}$$

On substituting for $W(\alpha,t)$ from Eq. (6) into Eq. (11), we obtain $\omega(x,\theta)$ for the amplified quantum state

$$\omega(x,\theta)=\sqrt{\frac{1}{\pi}}\frac{1}{\sqrt{(G-1)[N+1/2-|M|\cos(2\theta-\varphi)]}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}d^2\beta W(\beta,0)\exp\left[\frac{-2(x-\sqrt{G}(\beta_x\cos\theta+\beta_y\sin\theta))^2}{(G-1)[N+1/2-|M|\cos(2\theta-\varphi)]}\right]. \tag{12}$$

Equation (12) indicates a one-to-one correspondence between the phase φ of the atomic coherence and the phase θ of the field quadrature. In order to reconstruct the Wigner function of the initial quantum state, we need a set of distribution function $\omega(x,\theta)$ for different values of θ varying from 0 to π.

The Wigner function can be constructed by amplifying the signal such that there is no noise in the desired quadrature and all the noise is fed into the conjugate quadrature. It follows from Eq. (12) that an amplified signal without added noise in the quadrature $x(\theta)$ can be obtained if we choose $2\theta-\varphi=0$. To obtain the noise free amplification, we prepare the atoms in a coherent superposition of levels $|a\rangle$ and $|c\rangle$ with a particular phase φ. The atoms are then injected inside the cavity where they amplify the initial quantum state. The noise free quadrature can be obtained by adjusting the phase of the local oscillator θ such that $\theta=\varphi/2$. To find the complete set of distributions $\omega(x,\theta)$, we prepare the amplifier for a set of values of atomic coherent superposition phases φ ranging from 0 to 2π and obtain noise free amplification for the desired quadratures. The Wigner function can then be reconstructed from the measured values of $\omega(x,\theta)$.

The noise-free quadrature distribution is given by

$$\omega(x,\theta)=\sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{(G-1)\exp(-2r)}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}d^2\beta W(\beta,0)\exp\left(\frac{-2(x-\sqrt{G}[\beta_x\cos\theta+\beta_y\sin\theta])^2}{(G-1)\exp(-2r)}\right), \tag{13}$$

where we have used

$$(G-1)(N+1/2-|M|)=1/2(G-1)\exp(-2r). \tag{14}$$

Here r is the squeezing parameter.

Once the quadrature distributions of the amplified signal are measured in balanced homodyne measurement, then the complete Wigner function is determined by carrying out the inverse Radon transformation familiar in tomographic imaging [11].

$$W(\alpha_x,\alpha_y)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\pi}\omega(x,\theta)|\eta|\exp[i\eta(x-\alpha_x\cos\theta-\alpha_y\sin\theta)]dx\,d\eta\,d\theta. \tag{15}$$

In order to obtain the Wigner function of the amplified state, we substitute Eq. (13) into Eq. (15), and readily find $W(\alpha_x,\alpha_y)$ to be given by the following relation:

$$W(\alpha_x,\alpha_y)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\pi} d^2\beta d\eta d\theta W(\beta,0)|\eta| \times\exp\left(-\frac{(G-1)\exp(-2r)}{8}\eta^2-i\eta[(\alpha_x-\sqrt{G}\beta_x)\cos\theta+(\alpha_y-\sqrt{G}\beta_y)\sin\theta]\right). \tag{16}$$

For $G=1$, we obtain the Wigner function for the original state.

In terms of the rescaled variables $\alpha_x'=\alpha_x/\sqrt{G}$ and $\alpha_y'=\alpha_y/\sqrt{G}$, Eq. (16) reduces as

$$W(\alpha_x',\alpha_y')=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\pi} d^2\beta d\eta' d\theta W(\beta,0)|\eta'| \times\exp\left(-\frac{(1-1/G)\exp(-2r)}{8}\eta'^2-i\eta'[(\alpha_x'-\beta_x)\cos\theta+(\alpha_y'-\beta_y)\sin\theta]\right). \tag{17}$$

For sufficiently large squeezing, i.e., for $r\to\infty$, we obtain the same original state for any arbitrary value of the gain parameter $G>1$. This shows that the proposed scheme allows us to fully reconstruct the original quantum state after its amplification through a phase-sensitive linear amplifier. However, an appropriate rescaling of the measured distribution is required.

As an example, we consider the Schrödinger cat state, which is the superposition of two coherent states $|\xi_0\rangle$ and $|-\xi_0\rangle$, which are 180° out of phase with respect to each other,

$$\Psi_0=\sqrt{N}[|\xi_0\rangle+|-\xi_0\rangle], \tag{18}$$

where $N^{-1}=2[1+\exp(-2\xi_0^2)]$ is the constant of normalization and ξ_0 is taken as real for the sake of simplicity. The Wigner function $W(\beta,0)$ of this state is defined as [31]

$$W(\beta,0)=\frac{1}{\pi[1+\exp(-2\xi_0^2)]}\{\exp[-2(\beta_x-\xi_0)^2-2\beta_y^2]+\exp[-2(\beta_x+\xi_0)^2-2\beta_y^2]+2\exp(-2\beta_x^2-2\beta_y^2)\cos(4\xi_0\beta_y)\}. \tag{19}$$

The Wigner function of the amplified Schrödinger cat state can be obtained by using the expression for $W(\beta,0)$ in Eq. (17). In terms of the rescaled variables $\alpha_x'=\alpha_x/\sqrt{G}$ and $\alpha_y'=\alpha_y/\sqrt{G}$, it is given by the following:

$$W(\alpha_x',\alpha_y')=\frac{1}{8\pi^2[1+\exp(-2\xi_0^2)]}\int_{-\infty}^{+\infty}\int_0^{\pi} d\eta' d\theta|\eta'|\left\{\exp\left(\frac{-[1+(1-1/G)\exp(-2r)]}{8}\eta'^2 -i\eta'[(\alpha_x'+\xi_0)\cos\theta+\alpha_y'\sin\theta]\right)+\exp\left(\frac{-[1+(1-1/G)\exp(-2r)]}{8}\eta'^2-i\eta'[(\alpha_x'-\xi_0)\cos\theta+\alpha_y'\sin\theta]\right) +\exp(-2\xi_0^2)\times\left[\exp\left(\frac{-[1+(1-1/G)\exp(-2r)]}{8}\eta'^2-i\eta'[\alpha_x'\cos\theta+(\alpha_y'+i\xi_0)\sin\theta]\right) +\exp\left(\frac{-[1+(1-1/G)\exp(-2r)]}{8}\eta'^2-i\eta'[\alpha_x'\cos\theta+(\alpha_y'-i\xi_0)\sin\theta]\right)\right]\right\}. \tag{20}$$

It is clear that for sufficiently large squeezing, i.e., $r\to\infty$, and for any arbitrary value of the gain parameter $G>1$, we obtain the Wigner function $W(\alpha_x',\alpha_y')$ for the initial Schrödinger cat state which is quite interesting. In the next section, we present the results of our numerical simulation for different values of the squeezing parameter r and for $G=1$ and 10.

III. RESULTS AND DISCUSSION

Here we present the results obtained after integrating Eq. (20). In Fig. 2(a), we show the plots of Wigner function for $\xi_0=2$ and $G=1$. The figure clearly shows two Gaussian hills at $\alpha_x=\pm 2$, which is the location of two coherent states and oscillations on the conjugate axis due to the superposition of two coherent states. This is the well known behavior

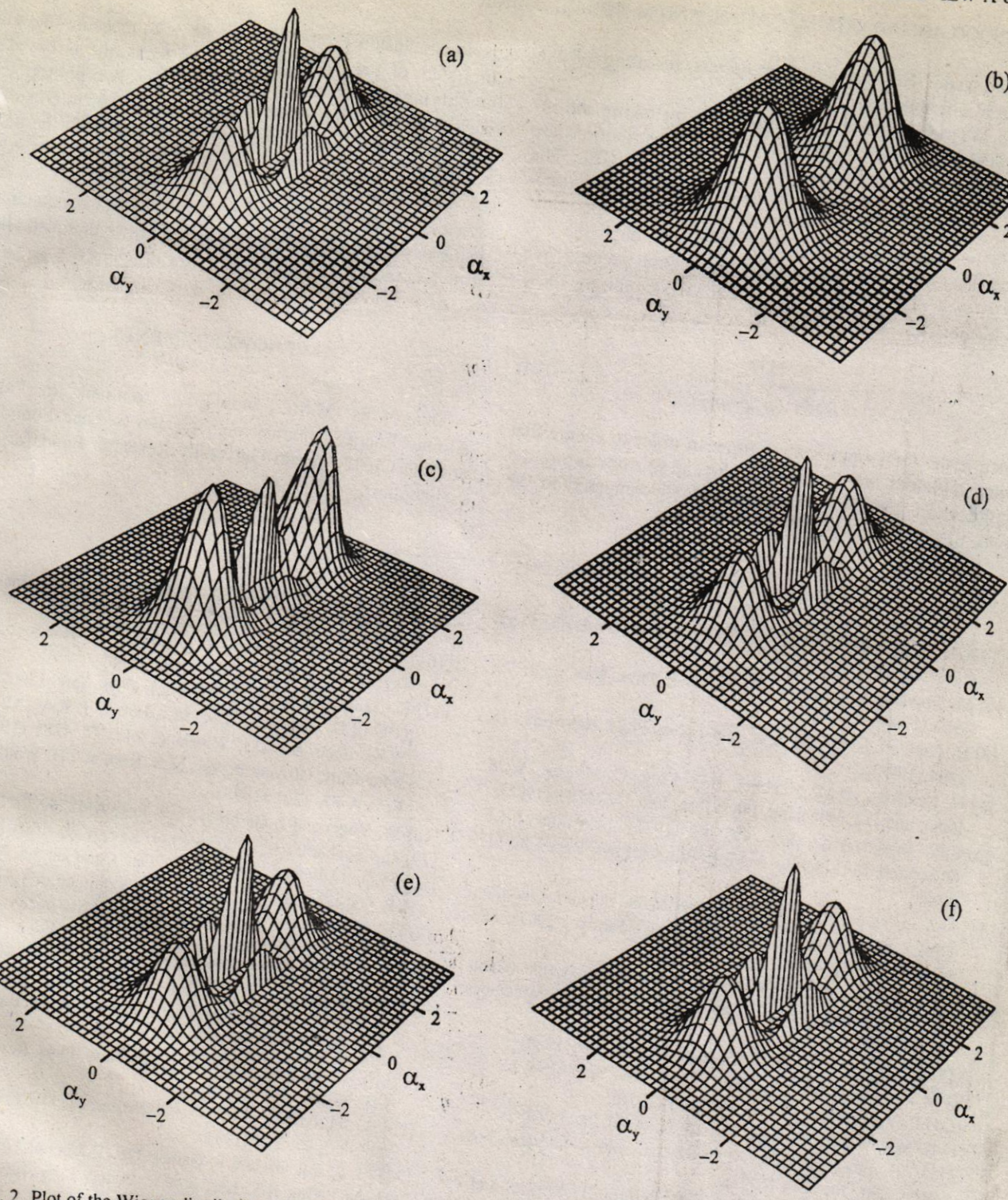

FIG. 2. Plot of the Wigner distribution $W(\alpha_x, \alpha_y)$ for the Schrödinger cat state. (a) For $\xi_0 = 2$ and $G = 1$. Here we obtain the well known structure associated with the Wigner function for Schrödinger cat state. In (b)–(f), we show the plots of the Wigner function for $\xi_0 = 2$. $G = 10, r = 0$ (phase-insensitive amplifier), and $r = 1$, 2, 3 and 4, respectively. The plots clearly show that Wigner function of the initial state almost fully recovered with the increase in the squeezing parameter r.

associated with the Schrödinger cat state. In Figs. 2(b)–2(f), we plot the Wigner function for $\xi_0 = 2$, $G = 10$, and $r = 0, 1, 2, 3$, and 4, respectively. Figure 2(b) clearly shows that the well known oscillations due to the Schrödinger cat state vanish when it is amplified through a phase insensitive amplifier. However, for $r = 1$ and 2 [see Figs. 2(c) and 2(d)] the oscillations start appearing which is quite interesting. For strong enough squeezing, i.e., $r = 3$ and $r = 4$, we almost fully recover the Wigner function corresponding to initial Schrödinger cat state. These results confirm our assertion that amplifying the signal with the help of a phase-sensitive linear amplifier allows us to fully reconstruct the original

quantum state. However, an appropriate rescaling of the measured distribution is needed.

The Wigner function is reconstructed by taking the inverse Radon transform, once the quadrature distributions are measured after amplification through two-photon CEL. The quadrature distributions can be measured using balanced homodyne detection scheme. The parameters in the experiment should be adjusted such that field leakage through the end mirror does not occur during the amplification process. We have $A = Rg^2\tau^2(\rho_{aa} - \rho_{cc})$ and $G = \exp(At)$, combining these two we obtain

$$t = \frac{\ln G}{Rg^2\tau^2(\rho_{aa} - \rho_{cc})}, \tag{21}$$

which is the total amplification time. In order to ensure that cavity field does not leak through the end mirrors during the amplification, the time t should be small compared to the cavity life time τ_c, i.e., $t \ll \tau_c$.

In conclusion, we propose a scheme to measure the quantum state of the radiation field. The technique is based on amplifying the signal with the help of a two-photon CEL such that there is no noise in the quadrature of interest. Our scheme is insensitive to problems associated with the detector inefficiencies. In a recent paper, Lenohardt and Paul [32] have also proposed an interesting scheme based on antisqueezing the field with respect to the desired quadrature using degenerate optical parameter amplifier that also allows to overcome the problem of detector efficiency.

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Abstract

We propose a method to measure the photon statistics of a quantized radiation field in an electromagnetically induced transparency setup. The proposed method provides a direct way of measuring the photon statistics. This method is insensitive to the detector efficiency. © 1998 Elsevier Science B.V.

Electromagnetically induced transparency (EIT) was first observed by Harris [1,2]. Since then this effect has been studied intensively both theoretically and experimentally [3–5]. The theoretical studies assume the driving field to be classical. In this paper we study EIT by a quantized driving field inside a cavity and show that the absorption spectrum provides a direct means of measuring the photon statistics of the field. This method of measuring the photon statistics has the advantage that the photon statistics of the radiation field can be directly measured from the spectrum without resorting to cumbersome numerical manipulations of the experimental data. In addition, the proposed method is insensitive to the detector efficiency which poses serious problems in observing nonclassical characteristics of the field.

The quantum state of the radiation field is described completely by the state vector $|\psi\rangle$ for a pure state and by the density operator ρ for a mixed state. The diagonal elements of the density operator with respect to the Fock state gives the photon distribution function. The photon distribution for many fields may demonstrate novel nonclassical features such as an oscillatory behavior in the case of single-mode squeezed vacuum state [6] or the Schrödinger-cat state [7]. It is a problem of recent interest to experimentally observe such nonclassical features of the quantum state of the radiation field.

The quantum state of the field is also determined by using optical homodyne tomography [8–10], which uses measured distributions of electric field quadrature amplitude to determine the Wigner function and hence the density matrix. From the knowledge of the density matrix, information about photon number and phase distributions is obtained. It has also been realized experimentally [11]. Other schemes include methods based on dispersive atom–field coupling in a Ramsey method of separated oscillatory fields [12], atomic beam deflection [13], the conditional measurements on the atoms in a micromaser set-up [14], the Autler–Townes spectroscopy [15], resonance fluorescence [16], homodyning [17], unbalanced homodyning [18], photon chopping [19], and photon counting [20].

In this paper, we propose a scheme to determine the photon statistics of the radiation field inside a cavity

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using a set-up that is employed in the observation of electromagnetically induced transparency (EIT). In EIT, a three-level atomic system is considered. When the upper levels are driven by a classical field, the medium becomes transparent for a probe field resonant with the lower level transition [21]. The transparency results from the combined Stark splitting and quantum interference of the dressed states which are created by applying that additional electromagnetic field. The splitting of the level is proportional to the associated Rabi frequency. Heights of the peaks of the absorption spectrum are independent of the Rabi frequency. Peaks are displaced from resonance by an amount equal to the Rabi frequency. If the upper levels are being driven by a quantized field, the associated Rabi frequencies are distributed according to the photon distribution of the driving field. The absorption spectrum would thus mimic the photon distribution function of the driving field which can therefore be recovered from the spectrum. The condition under which the photon distribution function of the driving field could be recovered is that the associated vacuum Rabi frequency should be larger than the atomic decay rates.

The method to determine the photon statistics based on Autler–Townes spectroscopy [15] is closely related to the one discussed in this paper. Another closely related scheme which has been experimentally realized for the determination of photon statistics is that of quantum Rabi oscillation [22]. Rabi oscillations have been observed in vacuum and in small coherent fields. Its Fourier components show the discrete nature of the field and the weighted Fourier components yield the photon number distribution in the field.

We consider a system of three-level atoms (see Fig. 1) initially in the ground state $|b\rangle$ interacting with a quantized radiation field inside a cavity. The upper levels $|a\rangle$ and $|c\rangle$ of the atom are driven by the cavity field which is quantized. We are interested in finding the photon statistics of the field. This is done by probing the absorption spectrum of the $|b\rangle$–$|a\rangle$ transition via a classical probe field of frequency ν. The decay rates from levels $|a\rangle$ and $|c\rangle$ are assumed to be γ_a and γ_c, respectively. The Hamiltonian for this system, in the dipole approximation and the rotating-wave approximation, is given by

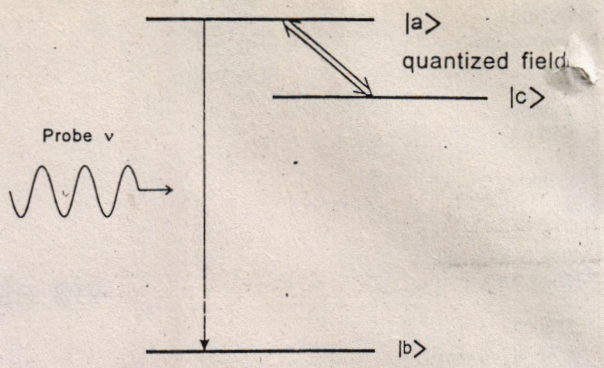

Fig. 1. Level scheme for the atom.

$$H = \hbar \sum_i \omega_i |i\rangle\langle i| + \hbar\nu a^\dagger a + \hbar g(|a\rangle\langle c|a + \text{c.c.}) - \tfrac{1}{2}(\wp_{ab}\varepsilon|a\rangle\langle b|e^{-i\nu t} + \text{c.c.}), \quad (1)$$

where $i = a, b, c$ represents the three atomic levels with ω_i being the transition frequency from the respective levels, the coupling constant g is the vacuum Rabi frequency between the levels $|a\rangle$ and $|c\rangle$, a and $a^\dagger$ are the annihilation and creation operators of the cavity field, ε is the amplitude of the probe field, and the corresponding dipole transition matrix element is represented by $\wp_{ab}$.

As the atoms are prepared initially in the ground state $|b\rangle$, we have $\rho^{(0)}_{bn,bn} = \rho_{n,n}$ and $\rho^{(0)}_{an,an} = \rho^{(0)}_{cn+1,cn+1} = 0$. We now show that the photon statistics of the radiation field inside the cavity can be determined by looking at the absorption spectrum at the $|b\rangle$–$|a\rangle$ transition.

The polarization of the medium is given by

$$P = \epsilon_0 \chi \varepsilon = \wp_{ab}[\rho_{ab} + \text{c.c.}]. \quad (2)$$

Here χ is the linear susceptibility whose real part is related to dispersion and the imaginary part gives the absorption spectrum. It is clear from this equation that, in order to find the susceptibility, we first need to determine the matrix element $\rho_{an,bn}$. A sum over n would give ρ_{ab}, and hence χ.

The matrix elements $\rho_{an,bn}$ and $\rho_{cn+1,bn}$ form the following coupled set of differential equations,

$$\frac{d\rho_{an,bn}}{dt} = -(i\omega_{ab} + \tfrac{1}{2}\gamma_a)\rho_{an,bn} - i(\wp_{ab}\varepsilon/2\hbar)e^{-i\nu t}(\rho_{an,an} - \rho_{bn,bn}) - ig\sqrt{n+1}\,\rho_{cn+1,bn}, \quad (3)$$

$$\frac{d\rho_{cn+1,bn}}{dt} = -(i\omega_{ab} + \tfrac{1}{2}\gamma_c)\rho_{cn+1,bn} - i(\wp_{ab}\varepsilon/2\hbar)e^{-i\nu t}\rho_{cn+1,an} - ig\sqrt{n+1}\,\rho_{an,bn}. \quad (4)$$

As we are interested in the linear susceptibility, we retain the matrix elements $\rho_{an,an}$, $\rho_{bn,bn}$, and $\rho_{cn+1,an}$ on the left-hand side up to zeroth order in the field amplitude ε. We therefore make the substitutions $\rho^{(0)}_{bn,bn} = \rho_{n,n}$ and $\rho^{(0)}_{an,an} = \rho^{(0)}_{cn+1,cn+1} = 0$. The slowly varying matrix elements $\tilde{\rho}_{\alpha n,\beta m}(t) = \rho_{\alpha n,\beta m}(t)\exp(i\nu t)$ then satisfy the following equations of motion,

$$\frac{d\tilde{\rho}_{an,bn_1}(t)}{dt} = -(i\delta + \tfrac{1}{2}\gamma_a)\tilde{\rho}_{an,bn_1}(t) + i(\wp_{ab}\varepsilon/2\hbar)\rho_{n,n} - ig\sqrt{n+1}\,\tilde{\rho}_{cn+1,bn_1}(t), \quad (5)$$

$$\frac{d\tilde{\rho}_{cn+1,bn_1}(t)}{dt} = -(i\delta + \tfrac{1}{2}\gamma_c)\tilde{\rho}_{cn+1,bn_1}(t) - ig\sqrt{n+1}\,\tilde{\rho}_{an,bn_1}(t), \quad (6)$$

where $\delta = \omega_{ab} - \nu$ is the detuning.

Eqs. (5) and (6) can be written in a compact form as

$$\dot{P}(t) = -MP(t) + Q, \quad (7)$$

where

$$P(t) = \begin{bmatrix} \tilde{\rho}_{an,bm}(t) \\ \tilde{\rho}_{cn+1,bm}(t) \end{bmatrix},$$

$$M = \begin{bmatrix} i\delta + \gamma_a/2 & ig\sqrt{n+1} \\ ig\sqrt{n+1} & i\delta + \gamma_c/2 \end{bmatrix},$$

$$Q = \begin{bmatrix} (i\wp_{ab}\varepsilon/2\hbar)\rho_{n,n} \\ 0 \end{bmatrix}. \quad (8)$$

A steady-state solution of Eq. (7) is given by

$$P(t) = M^{-1}Q. \quad (9)$$

The matrix element $\rho_{an,bn}$ can be determined from Eq. (9). It follows, on taking a trace over the field states, that

$$\rho_{ab}(t) = \frac{i\wp_{ab}\varepsilon}{2\hbar}\sum_n p(n) \times \frac{(i\delta + \gamma_c/2)e^{-i\nu t}}{(i\delta + \gamma_a/2)(i\delta + \gamma_c/2) + g^2(n+1)}, \quad (10)$$

where $p(n) \equiv \rho_{n,n}$ is the photon distribution function of the driving field inside the cavity.

The complex susceptibility χ of the medium can be determined using Eqs. (2) and (10). The real and imaginary parts of the complex susceptibility are

$$\chi'(\delta) = \frac{|\wp_{ab}|^2\delta}{4\epsilon_0\hbar}\sum_n p(n) \times \frac{\gamma_c(\gamma_a+\gamma_c)/2 + 2[\delta^2 - g^2(n+1) - \gamma_a\gamma_c/4]}{]\delta^2 - g^2(n+1) - \gamma_a\gamma_c/4]^2 + \delta^2(\gamma_a+\gamma_c)^2/4}. \quad (11)$$

$$\chi''(\delta) = \frac{|\wp_{ab}|^2}{4\epsilon_0\hbar}\sum_n p(n) \times \frac{\gamma_c[g^2(n+1) - \delta^2 + \gamma_a\gamma_c/4] + \delta^2(\gamma_a+\gamma_c)}{[\delta^2 - g^2(n+1) - \gamma_a\gamma_c/4]^2 + \delta^2(\gamma_a+\gamma_c)^2/4}. \quad (12)$$

For exact resonance ($\delta = 0$) and $\gamma_c \ll \gamma_a$, both the real and imaginary parts of the susceptibility vanish. The medium therefore becomes transparent. This is the condition of the electromagnetically induced transparency. This result is valid for arbitrary photon statistics of the driving field.

Eq. (12) can be rewritten as

$$\chi''(\delta) = \sum_n p(n)\,\chi''_n(\delta). \quad (13)$$

The function $\chi''_n(\delta)$ has double peaks located at $\delta = \pm g\sqrt{n+1}$. The height of both the peaks is proportional to $1/\gamma_a$. An important and interesting fact is that the height of the peaks is independent of the excitation number n. For a plot of $\chi''_n(\delta)$ versus $\delta^2 - g^2$, there is only one peak located at $g^2 n$.

Including the contributions from all the photon excitations in the photon distribution function $p(n)$ and in the limit that the decay rate γ_a is much less than the vacuum Rabi frequency g of the driving field, we get the complete absorption spectrum, as shown in Eq. (13). This absorption spectrum ($\chi''(\delta)$ versus δ^2) will mimic the photon distribution function $p(n)$.

We next illustrate our results by considering the example of a Schrödinger-cat state which is a coherent superposition of two coherent states, i.e.,

$$\psi = \frac{1}{\sqrt{\mathcal{N}}}(|\alpha\rangle + |-\alpha\rangle), \quad (14)$$

where

$$\mathcal{N} = 2(1 + e^{-2|\alpha|^2}) \quad (15)$$

of the medium can
d (10). The real and
susceptibility are

$$\frac{(n+1)-\gamma_a\gamma_c/4]}{^2+\delta^2(\gamma_a+\gamma_c)^2/4}, \quad (11)$$

$$\frac{/4]+\delta^2(\gamma_a+\gamma_c)}{^2+\delta^2(\gamma_a+\gamma_c)^2/4}. \quad (12)$$

d $\gamma_c \ll \gamma_a$, both the
susceptibility vanish.
transparent. This is
tically induced trans-
bitrary photon statis-

(13)

peaks located at $\delta =$
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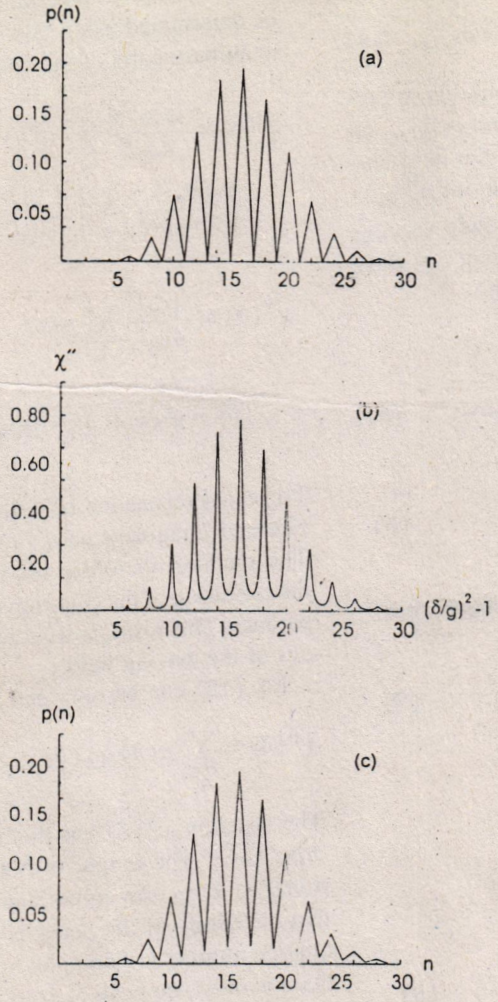

Fig. 2. (a) The photon distribution function of a Schrödinger-cat state $p(n)$ versus n at $\alpha = 4$. (b) The corresponding absorption spectrum χ'' (in arbitrary units) plotted against $\delta^2/g^2 - 1$ with $\gamma_a/g = 0.1$ and $\gamma_c/g = 0.0001$. (c) The recovered photon distribution from the spectrum χ''.

is the normalization constant. Here α is assumed to be real. The photon distribution function of the Schrödinger-cat state is given by

$$p(n) = 4e^{-|\alpha|^2}|\alpha|^{2n}/\mathcal{N}n!, \quad \text{when } n \text{ is even},$$
$$= 0, \quad \text{when } n \text{ is odd}, \quad (16)$$

The photon distribution is thus an oscillatory function of n. These oscillations are manifestations of nonclassical features of the quantum statistics.

In Fig. 2a, the photon distribution function $p(n)$ is plotted against n. The corresponding absorption spectrum $\chi''(\delta)$ versus $\delta^2/g^2 - 1$ is plotted in Fig. 2b. The photon distribution function, recovered from the absorption spectrum in the same way as mentioned above, is given in Fig. 2c.

This scheme for the measurement of photon statistics through EIT is feasible within the presently accessible experimental limits [23]. A small Fabry–Perot cavity, as reported by Hood et al. [24], where a single atom interacts with a cavity field, is appropriate for our scheme. Here $g = 60 \times 2\pi$ MHz, which is determined by the cavity geometry, and the atomic decay rate $\gamma = 2.6 \times 2\pi$ MHz. These values are in accordance with the condition required by our scheme that $g \gg \gamma$ in order to resolve the peaks of the photon distribution clearly. An improvement would be required as far as the cavity interaction time is concerned, which is small in this case, contrary to our requirement. In the microwave region, however, a large cavity interaction time τ_c has been observed in addition to the desired values of g and γ [25]. The values are $g = 17 \times 2\pi$ MHz, $\tau_c = 6 \times 2\pi$ kHz and $\gamma = 5 \times 2\pi$ Hz. One discrepancy, however, has not been encountered here; the values of g and γ referred to here are for the same two levels, which is not our case.

In this paper we have discussed a method based on absorption spectrum to measure the photon statistics of the radiation field using electromagnetically induced transparency. This is a conceptually simple and direct method and involves no cumbersome numerical inversions like that used in some other schemes for the same purpose. Another advantage of this method is that it is insensitive to the detector efficiency, which can create serious problems in the observation of nonclassical features of the quantum states.

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Quantum teleportation of an entangled state

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We consider the teleportation of entangled two-particle and multiparticle states and present a scheme for the teleportation that may be suitable for both entangled atomic states or field states inside high-Q cavities.

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I. INTRODUCTION

The notions of coherent superposition and entanglement in quantum mechanics lie at the conceptual foundation of quantum mechanics as evident through fundamental contributions like bell inequalities [1] and Greenberger-Horne-Zeilinger (GHZ) equalities [2]. These alternative concepts are finding interesting and useful applications in the field of quantum computing and quantum information.

One of the key problems in quantum communication is how to transmit a quantum state from one observer to another and keep the received state exactly the same as that sent without necessitating the movement of an information carrier. This can be accomplished in two steps. First, the sender "disassembles" information of a quantum state into two parts, one of which is sent through a quantum channel run by the nonlocal correlation between two entangled quantum entities, and the other of which is sent through a classical channel. Second, the receiver "reconstructs" the state on the basis of information from both the quantum and classical channels. Because in this process a quantum state to be transmitted is destroyed in one place and later reconstructed in another place, this transmission is termed as teleportation of a quantum state. Bennett *et al.* [3] proposed a scheme for the teleportation of an unknown quantum state from one observer to another through dual Einstein-Podolsky-Rosen (EPR) and classical channels.

Since this proposal was made, a number of experimentally feasible schemes have been suggested for the teleportation of two-level atomic states [4–13] and field states [14–16] for two-dimensional states to N-dimensional states [17]. Most of these schemes rely on methods based on cavity quantum electrodynamics in which two identical high-Q cavities are initially prepared in an entangled state. Quantum teleportation was experimentally verified by producing pairs of entangled photons through the process of parametric down-conversion [18]. Recently, a scheme has been presented that exploits the cavity decay for the teleportation of the atomic state of an atom trapped in a leaky cavity [19]. In addition to these schemes of discrete variables, much progress has also been made for the quantum teleportation of states of dynamical variables with continuous spectra [20–22]. The teleportation of a coherent state of the radiatic field [23] and the teleportation of a superposition of chir amplitudes have also been reported [24].

All these schemes are for the teleportation of single-qub states. In many potential applications of quantum computin such as factorizing a very large number [25] or searching a unordered quantum database [26], one needs the system c many-qubit states. It is therefore an interesting questio whether we can teleport a multiqubit state. In this paper, w present a scheme for the teleportation of a two-particle (two qubit) entangled state from a pair of high-Q cavities to an other pair of high-Q cavities using methods based on cavit quantum electrodynamics. This scheme is then generalize for the teleportation of the N-qubit field state.

II. QUANTUM TELEPORTATION OF AN ENTANGLED STATE

In this section, we consider the teleportation of a two-qubit entangled state of the radiation field in two separated high-Q cavities A_1 and A_2 to another pair of high-Q cavities C_1 and C_2. The entangled state of the radiation field is assumed to be

$$|\psi(A_1,A_2)\rangle=\sum_{p_1,p_2=0}^{1} C_{p_1p_2}|p_1,p_2\rangle. \tag{1}$$

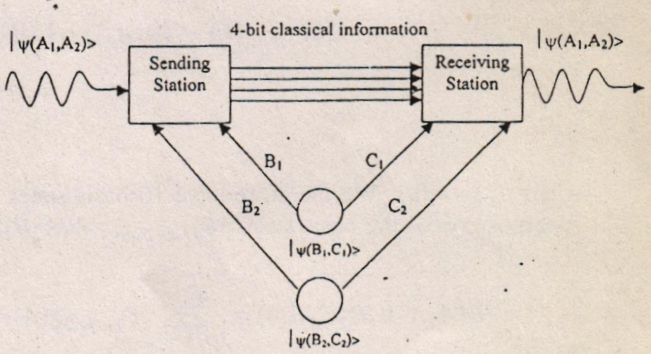

FIG. 1. Quantum teleportation of the two-qubit state $|\psi(A_1A_2)\rangle=\Sigma_{n_1,n_2=0}^{1}C_{n_1,n_2}|n_1,n_2\rangle$. $|\psi(B_1C_1)\rangle$ and $|\psi(B_2C_2)\rangle$ are two entangled states. Cavities B_1 and B_2 belong to the sending station while cavities C_1 and C_2 belong to the receiving station. A four-bit piece of classical information transmitted from the sending station to the receiving station enables the receiver to reconstruct the original state.

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It may be pointed out that this scheme also corresponds to the teleportation of entangled two-level atomic states because the atomic entanglement can be transferred to the two cavities by passing them through the two cavities with π pulse. As usual, the teleportation of state (1) can be carried out in three steps, as shown in Fig. 1.

In the first step, we consider the other two sets of cavities, B_1, C_1 and B_2, C_2, prepared in entangled states:

$$|\psi(B_1C_1)\rangle=\frac{1}{\sqrt{2}}(|0_{B_1},1_{C_1}\rangle+|1_{B_1},0_{C_1}\rangle), \tag{2}$$

$$|\psi(B_2C_2)\rangle=\frac{1}{\sqrt{2}}(|0_{B_2},1_{C_2}\rangle+|1_{B_2},0_{C_2}\rangle). \tag{3}$$

We then have

$$|\psi(B_1B_2C_1C_2)\rangle=\frac{1}{2}[|0_{B_1},0_{B_2},1_{C_1},1_{C_2}\rangle+|0_{B_1},1_{B_2},1_{C_1},0_{C_2}\rangle+|1_{B_1},0_{B_2},0_{C_1},1_{C_2}\rangle+|1_{B_1},1_{B_2},0_{C_1},0_{C_2}\rangle]. \tag{4}$$

It is important to note here that for the teleportation of a two-qubit quantum state we do not need to prepare an entangled state of four qubits. Instead, we need two entangled states of two qubits each. The combined state of the fields in the cavities A_1, A_2, B_1, B_2, C_1, and C_2 is therefore given as

$$|\psi(A_1A_2B_1B_2C_1C_2)\rangle=\tfrac{1}{2}\sum_{p_1,p_2=0}^{1}C_{p_1p_2}|p_1\rangle_{A_1}|p_2\rangle_{A_2}\times(|0_{B_1},0_{B_2},1_{C_1},1_{C_2}\rangle+|0_{B_1},1_{B_2},1_{C_1},0_{C_2}\rangle+|1_{B_1},0_{B_2},0_{C_1},1_{C_2}\rangle+|1_{B_1},1_{B_2},0_{C_1},0_{C_2}\rangle). \tag{5}$$

Next we define the basis states for the $A_1A_2B_1B_2$ system:

$$|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_1},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},1_{B_1},0_{B_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle), \tag{6}$$

$$|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_1},0_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},1_{B_1},1_{B_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},0_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_1},1_{B_2}\rangle), \tag{7}$$

$$|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_1},1_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},1_{B_1},0_{B_2}\rangle), \tag{8}$$

$$|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},0_{B_1},0_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_1},1_{B_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_1},0_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},1_{B_1},1_{B_2}\rangle), \tag{9}$$

where $j_1,j_2=0,1$. We therefore have 16 basis states. The combined state $|\psi(A_1A_2B_1B_2C_1C_2)\rangle$ can be rewritten as a linear superposition of the basis states $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ of the $A_1A_2B_1B_2$ system as follows:

$$\begin{aligned}|\psi(A_1A_2B_1B_2C_1C_2)\rangle=\sum_{j_1,j_2=0}^{1}&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle(C_{00}|0_{C_1},0_{C_2}\rangle+C_{01}e^{i\pi j_2}|0_{C_1},1_{C_2}\rangle+C_{10}e^{i\pi j_1}|1_{C_1},0_{C_2}\rangle\\&+C_{11}e^{i\pi(j_1+j_2)}|1_{C_1},1_{C_2}\rangle)+|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle(C_{00}|0_{C_1},1_{C_2}\rangle+C_{01}e^{i\pi j_2}|0_{C_1},0_{C_2}\rangle\\&+C_{10}e^{i\pi j_1}|1_{C_1},1_{C_2}\rangle+C_{11}e^{i\pi(j_1+j_2)}|1_{C_1},0_{C_2}\rangle)\\&+|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle(C_{00}|1_{C_1},0_{C_2}\rangle+C_{01}e^{i\pi j_2}|1_{C_1},1_{C_2}\rangle+C_{10}e^{i\pi j_1}|0_{C_1},0_{C_2}\rangle\\&+C_{11}e^{i\pi(j_1+j_2)}|0_{C_1},1_{C_2}\rangle)+|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle(C_{00}|1_{C_1},1_{C_2}\rangle+C_{01}e^{i\pi j_2}|1_{C_1},0_{C_2}\rangle\\&+C_{10}e^{i\pi j_1}|0_{C_1},1_{C_2}\rangle+C_{11}e^{i\pi(j_1+j_2)}|0_{C_1},0_{C_2}\rangle).\end{aligned} \tag{10}$$

In the second step, we make a measurement of the $A_1A_2B_1B_2$ system. A detection of the $A_1A_2B_1B_2$ system in the state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ projects the field state in the cavities C_1C_2 into

$$|\psi(C_1C_2)\rangle=\sum_{p_1,p_2=0}^{1} e^{i\pi(j_1p_1+j_2p_2)}C_{p_1p_2}|(k_1+p_1)\mathrm{mod}2\rangle_{C_1}\times|(k_2+p_2)\mathrm{mod}2\rangle_{C_2}. \quad (11)$$

The field state in the cavities C_1C_2 has thus been projected to a state that has all the information about the amplitudes $C_{p_1p_2}$.

In the third and final step of the quantum teleportation, a manipulation of the cavities C_1C_2 needs to be done to bring state (11) to from (i). In the following sections we give the details of these three steps.

A. Preparation of entangled states

In the first step, we prepare two pairs of cavities B_1,C_1 and B_2,C_2 in entangled states (2) and (3). This can be done by passing a two-level atom initially in the excited state through the two resonant cavities. The interaction times of an atom with two cavities are chosen to be such that we have a $\pi/2$ pulse in the first cavity and a π pulse in the second cavity [6]. Initially, the two cavities B_1 and C_1 are taken in a vacuum and the two-level atom is taken in an excited state $|a\rangle$. When the atom has undergone a $\pi/2$ pulse in the first cavity, the second cavity is still empty and the atom-field system is in a state that corresponds to a linear superposition with equal weights of $|a\rangle$ and $|b\rangle$ atomic states correlated to zero and one photon, respectively, in cavity B_1 as

$$|\psi(B_1C_1)\rangle=\frac{1}{\sqrt{2}}(|b,1_{B_1}\rangle+|a,0_{B_1}\rangle)\otimes|0_{C_1}\rangle. \quad (12)$$

If the atom is still in an excited state $|a\rangle$ after leaving cavity B_1 in its vacuum state, it will, with unit probability, be transferred to $|b\rangle$ by the π pulse in cavity C_1 and leave a photon in the second cavity. If it emits a photon in cavity B_1 and exits it in level $|b\rangle$, it will be unaffected by its coupling with the vacuum in cavity C_1 and will leave the second cavity empty. Thus the atom always exits from second cavity C_1 in state $|b\rangle$, while the field is left in the entangled state (2). Similarly, we prepare another pair of cavities, B_2C_2, in entangled state (3).

B. Measurement of $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$

The second step of the teleportation is the measurement of the Bell states $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ of the $A_1A_2B_1B_2$ system. The subscripts j_1, j_2, k_1, and k_2 have the values 0 and 1. Among these, k_1 and k_2 can be determined by the number of photons inside the four cavities, while j_1 and j_2 can be determined from the relative phase of the states. Thus the state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ can be determined in two sets of measurements, the first determining k_1 and k_2 via the total number of photons inside the cavities, and the second determining j_1 and j_2 via the relative phase. It is clear that the cavities in state $|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle$ have two photons, those in states $|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle$ and $|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle$ have one or three photons, while in state $|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle$ they have zero, two, or four photons.

There are a number of ways to determine the number of photons inside the cavities. We propose the use of Ramsey interferometry. In this scheme, we consider two-level atoms initially prepared in ground state $|b\rangle$ that are off-resonant with the radiation field inside the cavities. The cavities are placed between two classical microwave fields (Ramsey zones R_1 and R_2) driving the $|a\rangle\rightarrow|b\rangle$ transition. When an atom passes from the first zone R_1 with a microwave field tuned at frequency ω_{ab}, it is prepared in a coherent superposition of states $(|a\rangle+|b\rangle)/\sqrt{2}$. This atom is then passed through the two selected cavities with the same interaction time $\hat{v}$ in each cavity. During the passage through the cavities, a phase shift proportional to the photon number s in the two cavities is introduced as a phase of the state $|b\rangle$ [27]. The resulting state of the atom then becomes

$$\frac{1}{\sqrt{2}}[|a\rangle+e^{is\theta}|b\rangle]. \quad (13)$$

The atom is then passed through the second zone R_2, again resonant with ω_{ab}. The interaction time and the coupling parameters are chosen such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$. The final atomic state is

$$e^{is\theta/2}[\cos(s\theta/2)|a\rangle-i\sin(s\theta/2)|b\rangle]. \quad (14)$$

The complete atom-field state is entangled and rather complicated. We have therefore not reproduced it here. It is, however, clear that a measurement of the atom in state $|a\rangle$ or $|b\rangle$ would reduce the fields inside the cavities to states with only an appropriate number of total photons in the two cavities.

The first atom is sent through the two cavities A_1 and B_1 with the interaction time $\theta=\pi$ in each cavity. It follows from Eq. (13) that if the atom is found in the excited state $|a\rangle$, the total number of photons in the two cavities is even, i.e., 0, 2. This implies $k_1=1$, $k_2=0$ or $k_1=1$, $k_2=1$. If the atom is detected in state $|b\rangle$, then the total number of photons in the two cavities is odd and $k_1=0$, $k_2=0$ or $k_1=0$, $k_2=1$. In the next step we make a measurement in the cavities A_2 and B_2 only with the same interaction time. A detection of an atom in either the excited state $|a\rangle$ or the ground state $|b\rangle$ completely determines the values of k_1 and k_2 according to the following sequence:

$$|a\rangle|a\rangle\Rightarrow|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle\Rightarrow|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle\Rightarrow|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle \Rightarrow |\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle.$$

For the determination of phase factors j_1 and j_2 we make measurements in the cavities A_1 and A_2 only after first evacuating the cavities B_1 and B_2. However, during the process of "emptying" the cavities B_1 and B_2, the relative phase between the component states in the resulting state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ may change. There are a number of ways to remove the photons from the cavities B_1 and B_2.

Here we consider two two-level atoms initially in their ground states $|b\rangle$. One of the atoms is sent through the cavity B_1 and the other through cavity B_2. After the passage, the atomic internal states $|a\rangle$ and $|b\rangle$ are mixed by a classical field such that $|a\rangle \rightarrow (|a\rangle + |b\rangle)/\sqrt{2}$ and $|b\rangle \rightarrow (|a\rangle - |b\rangle)/\sqrt{2}$. A subsequent detection of these atoms in states $|a\rangle$ or $|b\rangle$ introduces phase factors. To see this clearly, we take $k_1 = k_2 = 0$ for the sake of simplicity. Similar arguments will, however, apply for other values of k_1 and k_2. First we consider the passage of atoms through cavity B_1 only. The initial state is therefore

$$\begin{aligned}
&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle \otimes |\text{atom}\rangle \\
&= \tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_1},1_{B_2}\rangle + e^{i\pi j_2}|0_{A_1},1_{A_2},1_{B_1},0_{B_2}\rangle \\
&\quad + e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle \\
&\quad + e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle) \otimes |b\rangle. \qquad (15)
\end{aligned}$$

The removal of a photon from B_1 followed by mixing of the atomic levels by the classical field yields

$$\begin{aligned}
&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle \otimes |\text{atom}\rangle \\
&= \tfrac{1}{2}[(|0_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle \\
&\quad + e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle) \otimes \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle) \\
&\quad + (e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle \\
&\quad + e^{i\pi j_2}|1_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle) \otimes \frac{1}{\sqrt{2}}(|a\rangle - |b\rangle)]. \qquad (16)
\end{aligned}$$

The detection of the atom in level $|a\rangle$ gives

$$\begin{aligned}
&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle \\
&= \tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_2}\rangle + e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_2}\rangle + e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_2}\rangle \\
&\quad + e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_2}\rangle) \otimes |0_{B_1}\rangle, \qquad (17)
\end{aligned}$$

whereas the detection of the atom in level $|b\rangle$ gives

$$\begin{aligned}
&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle \\
&= \tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_2}\rangle + e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_2}\rangle \\
&\quad - e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_2}\rangle - e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_2}\rangle) \otimes |0_{B_1}\rangle. \qquad (18)
\end{aligned}$$

By a similar procedure, the photon can be removed from the cavity B_2 and the resulting cavity field state will have phase factors according to the final outcome of the atomic state. Here we summarize the final outcome depending upon the sequence of atom states for the removal of photons from cavity B_1 and cavity B_2:

$$\begin{aligned}
|a\rangle|a\rangle &\rightarrow \tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle + e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle + e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle \\
&\quad + e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle) \otimes |0_{B_1},0_{B_2}\rangle, \\
|a\rangle|b\rangle &\rightarrow \tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle - e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle + e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle \\
&\quad - e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle) \otimes |0_{B_1},0_{B_2}\rangle, \\
|b\rangle|a\rangle &\rightarrow \tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle + e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle - e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle \\
&\quad - e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle) \otimes |0_{B_1},0_{B_2}\rangle, \\
|b\rangle|b\rangle &\rightarrow \tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle - e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle - e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle \\
&\quad + e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle) \otimes |0_{B_1},0_{B_2}\rangle. \qquad (19)
\end{aligned}$$

This completes the procedure of evacuating cavities B_1 and B_2. The resulting state can have different but known phase factors between the constituent states. The net effect is equivalent to a transformation to a different basis. Next we make measurements in the cavities A_1 and A_2 in order to determine the phase factors j_1 and j_2. For simplicity's sake, we assume that the first two atoms are detected in state $|a\rangle$.

We now remove photons from cavities A_1 and A_2 by a similar procedure, i.e., by sending two-level atoms in their ground state $|b\rangle$ followed again by a classical field that mixes the levels such that $|a\rangle \rightarrow (|a\rangle + |b\rangle)/\sqrt{2}$ and $|b\rangle \rightarrow (|a\rangle - |b\rangle)/\sqrt{2}$:

$$\begin{aligned}
&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle \otimes |\text{atom}\rangle \\
&= \frac{1}{2\sqrt{2}}(|0_{A_2}\rangle + e^{i\pi j_2}|1_{A_2}\rangle) \\
&\quad \times [(1 + e^{i\pi j_1})|a\rangle - (1 - e^{i\pi j_1})|b\rangle] \otimes |0_{A_1},0_{B_1},0_{B_2}\rangle. \qquad (20)
\end{aligned}$$

If the atom is detected in $|a\rangle$ then $j_1 = 0$, and if atom is detected in $|b\rangle$ then $j_1 = 1$. The resulting cavity field state is

$$\begin{aligned}
|\psi_{j_2,0,0}(A_1A_2B_1B_2)\rangle &= \frac{1}{\sqrt{2}}(|0_{A_2}\rangle + e^{i\pi j_2}|1_{A_2}\rangle) \\
&\quad \otimes |0_{A_1},0_{B_1},0_{B_2}\rangle. \qquad (21)
\end{aligned}$$

Finally, we send the atom through the cavity A_2 and repeat the same procedure. If the atom is found in state $|a\rangle$ then $j_2=0$, and if the atom is detected in $|b\rangle$ then $j_2=1$. So by making measurements only in cavities A_1 and A_2 by first removing one photon from cavity A_1 and then removing one photon from cavity A_2, detection of the atom in different states yields the different values of j_1 and j_2 as

$$|a\rangle|a\rangle|a\rangle|a\rangle \rightarrow j_1=0, \quad j_2=0 \Rightarrow |\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|a\rangle|a\rangle|b\rangle \rightarrow j_1=0, \quad j_2=1 \Rightarrow |\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|a\rangle|b\rangle|a\rangle \rightarrow j_1=1, \quad j_2=0 \Rightarrow |\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|a\rangle|b\rangle|b\rangle \rightarrow j_1=1, \quad j_2=1 \Rightarrow |\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle.$$

If we have other sequences of detection of the first two atoms, then by doing the same process detection of the atom in different states gives the different values of j_1 and j_2 as shown below:

$$|a\rangle|b\rangle|a\rangle|a\rangle \rightarrow j_1=0, \quad j_2=1 \Rightarrow |\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle|a\rangle|b\rangle \rightarrow j_1=0, \quad j_2=0 \Rightarrow |\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle|b\rangle|a\rangle \rightarrow j_1=1, \quad j_2=1 \Rightarrow |\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle|b\rangle|b\rangle \rightarrow j_1=1, \quad j_2=0 \Rightarrow |\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|a\rangle|a\rangle \rightarrow j_1=1, \quad j_2=0 \Rightarrow |\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|a\rangle|b\rangle \rightarrow j_1=1, \quad j_2=1 \Rightarrow |\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|b\rangle|a\rangle \rightarrow j_1=0, \quad j_2=0 \Rightarrow |\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|b\rangle|b\rangle \rightarrow j_1=0, \quad j_2=1 \Rightarrow |\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|a\rangle|a\rangle \rightarrow j_1=1, \quad j_2=1 \Rightarrow |\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|a\rangle|b\rangle \rightarrow j_1=1, \quad j_2=0 \Rightarrow |\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|b\rangle|a\rangle \rightarrow j_1=0, \quad j_2=1 \Rightarrow |\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|b\rangle|b\rangle \rightarrow j_1=0, \quad j_2=0 \Rightarrow |\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle.$$

We can summarize from the above equations that if the order of detection of the first two atoms is the same as the last two, then we have $j_1=0$ and $j_2=0$ and the state is $|\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle$. If the detection of atomic states is the same for the first and third atom and detection of the fourth atom is reversed with respect to the second atom, then $j_1=0$ and $j_2=1$ and the state is $|\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle$. If the detection of the atomic states is the same for the second and fourth atom and detection of the third atom is reversed with respect to the first atom, then $j_1=1$ and $j_2=0$ and the state is $|\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle$. If the order of detection of atomic states for the third and fourth atom is reversed with respect to the first and second atom, respectively, then $j_1=1$ and $j_2=1$ and the state is $|\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle$.

A determination of the entangled state of the field inside the cavities A_1, A_2, B_1, and B_2, say, in state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$, projects the state of the field in cavities C_1 and C_2 into the state $|\psi(C_1C_2)\rangle$ as given by Eq. (11). In the final step of the teleportation, we transform this state into the original state (1).

C. Transformation

The transformation of state $|\psi(C_1C_2)\rangle$ given by Eq. (11) into that given by Eq. (1) involves two steps. One is the removal of phases $\exp(i\pi j_1)$ and $\exp(i\pi j_2)$ and the other is an appropriate transformation of photon numbers.

First we consider the transformation of phase only. For the sake of simplicity, we take $k_1=0$ and $k_2=0$. We then have

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}e^{i\pi j_2}|0_{C_1},1_{C_2}\rangle + C_{10}e^{i\pi j_1}|1_{C_1},0_{C_2}\rangle + C_{11}e^{i\pi(j_1+j_2)}|1_{C_1},1_{C_2}\rangle. \tag{22}$$

(i) If $j_1=0$ and $j_2=0$, then the state $|\psi(A_1A_2)\rangle$ is recovered.

(ii) If $j_1=0$ and $j_2=1$, then

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}e^{i\pi}|0_{C_1},1_{C_2}\rangle + C_{10}|1_{C_1},0_{C_2}\rangle + C_{11}e^{i\pi}|1_{C_1},1_{C_2}\rangle. \tag{23}$$

An atom in a superposition state $[|a\rangle+|b\rangle]/\sqrt{2}$ is passed through the cavity C_2 only in such a way that the ground state $|b\rangle$ picks the phase $\exp(ip\pi)$ (p being the number of photons inside the cavity C_2) while the excited state $|a\rangle$ does not pick any additional phase. We then have

$$|\psi(C_1C_2)\rangle = \frac{1}{\sqrt{2}}(C_{00}|0_{C_1},0_{C_2}\rangle - C_{01}|0_{C_1},1_{C_2}\rangle + C_{10}|1_{C_1},0_{C_2}\rangle - C_{11}|1_{C_1},1_{C_2}\rangle)|a\rangle + \frac{1}{\sqrt{2}}(C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}|0_{C_1},1_{C_2}\rangle + C_{10}|1_{C_1},0_{C_2}\rangle + C_{11}|1_{C_1},1_{C_2}\rangle)|b\rangle. \tag{24}$$

If the atom is detected in $|b\rangle$ after the passage through cavity C_2 then the state $|\psi(A_1A_2)\rangle$ is recovered. If the atom is detected in state $|a\rangle$ then repeat the process until the atom is detected in $|b\rangle$.

(iii) For $j_1=1$ and $j_2=0$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}|0_{C_1},1_{C_2}\rangle + C_{10}e^{i\pi}|1_{C_1},0_{C_2}\rangle + C_{11}e^{i\pi}|1_{C_1},1_{C_2}\rangle. \tag{25}$$

We carry out the same process again, but this time we pass the atom through cavity C_1 only. With the detection of the atom in state $|b\rangle$, we recover the required state.

(iv) For $j_1=1$ and $j_2=1$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}e^{i\pi}|0_{C_1},1_{C_2}\rangle + C_{10}e^{i\pi}|1_{C_1},0_{C_2}\rangle + C_{11}|1_{C_1},1_{C_2}\rangle. \quad (26)$$

Again, the same procedure is repeated except that the atom passes through both cavities. As before, the detection of the atom in state $|b\rangle$ will recover the required state; otherwise, we repeat the process until it is detected in state $|b\rangle$.

Next we consider the transformation of photon numbers in the cavities. As phase is removed by the method discussed above, we take $j_1=j_2=0$ for simplicity's sake:

(i) For $k_1=0$ and $k_2=0$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}|0_{C_1},1_{C_2}\rangle + C_{10}|1_{C_1},0_{C_2}\rangle + C_{11}|1_{C_1},1_{C_2}\rangle, \quad (27)$$

and the original state is recovered and we do nothing further.

(ii) For $k_1=0$ and $k_2=1$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},1_{C_2}\rangle + C_{01}|0_{C_1},0_{C_2}\rangle + C_{10}|1_{C_1},1_{C_2}\rangle + C_{11}|1_{C_1},0_{C_2}\rangle. \quad (28)$$

In order to recover the original state (1), we should interchange the state between zero and one photon in cavity C_2. For this purpose, we pass a two-level atom in its ground state $|b\rangle$ through cavity C_2 with a π pulse followed by its passage through a classical field again with a π pulse ($|a\rangle\rightarrow|b\rangle$ and $|b\rangle\rightarrow|a\rangle$) and finally through an empty cavity C_2' such that the atom in excited state $|a\rangle$ leaves the cavity in ground state $|b\rangle$ while leaving one photon inside the cavity and the atom in ground state $|b\rangle$ leaves the cavity in the ground state with no photon inside the cavity. This leads to the field states in the cavities C_1 and C_2' in the entangled state (1) and the teleportation is complete.

(iii) For $k_1=1$ and $k_2=0$,

$$|\psi(C_1C_2)\rangle = C_{00}|1_{C_1},0_{C_2}\rangle + C_{01}|1_{C_1},1_{C_2}\rangle + C_{10}|0_{C_1},0_{C_2}\rangle + C_{11}|0_{C_1},1_{C_2}\rangle. \quad (29)$$

We carry out the same procedure as above with the only difference being that the atom is passed through cavity C_1.

(iv) For $k_1=1$ and $k_2=1$,

$$|\psi(C_1C_2)\rangle = C_{00}|1_{C_1},1_{C_2}\rangle + C_{01}|1_{C_1},0_{C_2}\rangle + C_{10}|0_{C_1},1_{C_2}\rangle + C_{11}|0_{C_1},0_{C_2}\rangle. \quad (30)$$

Here we carry out the above procedure independently for the two cavities C_1 and C_2.

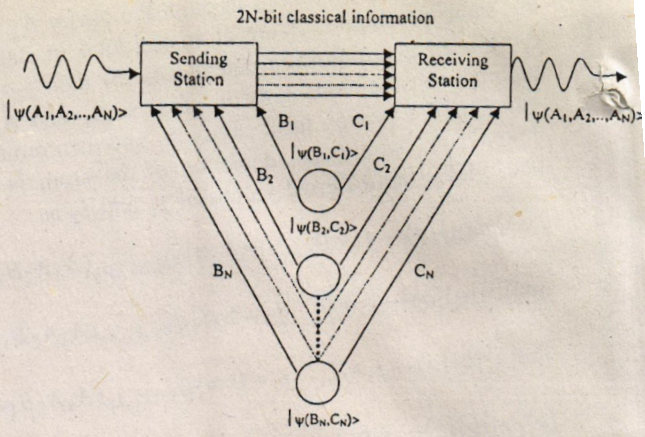

FIG. 2. Quantum teleportation of the N-qubit state $|\psi(A_1A_2\ldots A_N)\rangle=\Sigma_{n_1,n_2,\ldots,n_N=0}^{1}C_{n_1,n_2,\ldots,n_N}|n_1,n_2,\ldots,n_N\rangle$. $|\psi(B_iC_i)\rangle$ are N entangled states. Cavities B_i $(i=1,2,\ldots,N)$ belong to sender while cavities C_i $(i=1,2,\ldots,N)$ are with the receiver. A $2N$-bit piece of classical information transmitted from the sending station to the receiving station enables the receiver to reconstruct the original state.

III. TELEPORTATION OF THE N-QUBIT FIELD STATE

After giving a scheme to teleport the two-qubit state, we would like to generalize this scheme for the N-qubit state as shown in Fig. 2. Let us consider a N-qubit entangled field state in N high-Q cavities as

$$|\psi(A_1\ldots A_N)\rangle = \sum_{n_1,\ldots,n_N=0}^{1} C_{n_1,\ldots,n_N}|n_1,\ldots,n_N\rangle. \quad (31)$$

We want to teleport this entangled state in A_i $(i=1,2,\cdots,N)$ high-Q cavities to C_i $(i=1,2,\ldots,N)$ high-Q cavities.

In the first step of the teleportation of state (31), we need N pairs of entangled cavities

$$|\psi(B_iC_i)\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{B_i}|1\rangle_{C_i}+|1\rangle_{B_i}|0\rangle_{C_i}), \quad (32)$$

where $i=1,2,\ldots,N$. These N entangled pairs of cavities can be prepared as mentioned earlier by passing two-level atoms initially in the excited state through the two resonant cavities and by setting a $\pi/2$ pulse and a π pulse, respectively, in the two cavities. As before, cavities B_i $(i=1,2,\ldots,N)$ are with the sender and cavities C_i $(i=1,2,\ldots,N)$ belong to receiver. We now define 2^{2N} basis states in cavities $A_1A_2\ldots A_NB_1B_2\ldots B_N$ as

$$|\psi_{j_1,\ldots,j_N,k_2,\ldots,k_N}(A_1\ldots A_NB_1\ldots B_N)\rangle = \sum_{p_1,\ldots,p_N=0}^{1} \exp[i\pi(j_1p_1+j_2p_2+\cdots+j_Np_N)] \times|p_1\rangle_{A_1}|p_2\rangle_{A_2}\cdots|p_N\rangle_{A_N}|(1-p_1-k_1)\mathrm{mod}2\rangle_{B_1} \times|(1-p_2-k_2)\mathrm{mod}2\rangle_{B_2}\times\cdots\times|(1-p_N-k_N)\mathrm{mod}2\rangle_{B_N}$$

$$= \sum_{p_1,\ldots,p_N=0}^{1} \prod_{m=1}^{N} [e^{i\pi j_m p_m}|p_m\rangle_{A_m} \times |(1-p_m-k_m)\mathrm{mod}2\rangle_{B_m}]. \quad (33)$$

The combined state in the cavities $A_1 \ldots A_N B_1 \ldots B_N C_1 \ldots C_N$ in terms of basis states can be written as

$$|\psi(A_1 \ldots A_N B_1 \ldots B_N C_1 \ldots C_N)\rangle = \sum_{j_1,\ldots,j_N=0}^{1} \sum_{k_1,\ldots,k_N=0}^{1} \sum_{p_1,\ldots,p_N=0}^{1} \times C_{p_1,\ldots,p_N} |\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1 \ldots A_N B_1 \ldots B_N)\rangle \times \prod_{m=1}^{N} e^{i\pi j_m p_m} |(p_m+k_m)\mathrm{mod}2\rangle_{C_m}. \quad (34)$$

We now make measurement of the 2^{2N} basis states of the $A_1 \ldots A_N B_1 \ldots B_N$ system. It has $2N$ parameters; N parameters correspond to the phase, while the remaining N parameters correspond to the photon numbers inside the cavities $A_1,\ldots A_N, A_N, B_1,\ldots,B_N$. Thus the state $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1 \ldots A_N B_1 \ldots B_N)\rangle$ can be determined in two sets of measurements, the first determining $k_1,k_2,\ldots,k_N$ via the total number of photons inside the cavities, and the second determining $j_1,j_2,\ldots,j_N$ via the relative phase. For the determination of photon numbers we use Ramsey interferometry. We send an atom in ground state $|b\rangle$ through two cavities A_1 and B_1 and two Ramsey zones R_1 and R_2 with interaction time $\theta = \pi$ in each cavity. The atom is resonant with the two Ramsey zones and off-resonant with the cavities. Detection of the atom in either the excited state $|a\rangle$ or the ground state $|b\rangle$ makes the probable outcomes of $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1 \ldots A_N B_1 \ldots B_N)\rangle$ to $N/2$ of total N values. We then send a second atom in the ground state through A_2 and B_2 with the same interaction time, which reduces the probable outcomes by half. Similarly, we continue the procedure and send the last atom through A_N and B_N. A detection of the atom in either the excited state $|a\rangle$ or the ground state $|b\rangle$ completely determines the values of $k_1,k_2,\ldots,k_N$ according to the following outcomes. For example, k_n is equal to 1 if the outcome of the nth atom is $|a\rangle$, and k_n is zero if the outcome of the nth atom is $|b\rangle$. For example, if the outcome of each atom is $|b\rangle$, except the last outcome, i.e., $|a\rangle$, then the Bell state is $|\psi_{j_1,\ldots,j_N,0,0,\ldots,1}(A_1 \ldots A_N B_1 \ldots B_N)\rangle$.

For the determination of phase factors $j_1,j_2,\ldots,j_N$ we make a measurement in the cavities $A_1,A_2,\ldots,A_N$ only after evacuating the cavities $B_1,B_2,\ldots,B_N$. For this purpose we follow the same procedure used earlier for the two-qubit state. We send N two-level atoms initially in ground state $|b\rangle$ one by one through the cavities $B_1,B_2,\ldots,B_N$. After the passage through the cavity, the atomic internal states $|a\rangle$ and $|b\rangle$ are mixed by a classical field such that $|a\rangle \rightarrow (|a\rangle + |b\rangle)/\sqrt{2}$ and $|b\rangle \rightarrow (|a\rangle - |b\rangle)/\sqrt{2}$. A subsequent detection of these atoms in state $|a\rangle$ or $|b\rangle$ introduces phase factors yielding 2^N possible outcomes of atomic states. Next we make measurements in $A_1,A_2,\ldots,A_N$ in order to determine $j_1,j_2,\ldots,j_N$. We remove one photon from A_1 by sending a two-level atom in its ground state $|b\rangle$ followed again by a classical field that mixes the levels such that $|a\rangle \rightarrow (|a\rangle + |b\rangle)/\sqrt{2}$ and $|b\rangle \rightarrow (|a\rangle - |b\rangle)/\sqrt{2}$. Detection of the atom in $|a\rangle$ or $|b\rangle$ determines the value of j_1. It is zero if the atom is detected in $|a\rangle$ and one if the atom is detected in $|b\rangle$. We then repeat the process with other cavities. Finally we send the Nth atom in $|b\rangle$ from cavity A_N, after mixing and detection of the atom in $|a\rangle$ or $|b\rangle$ determines j_N. For each combination of the first step, while evacuating $B_1,B_2,\ldots,B_N$, we get 2^N combinations in the second step. Finally, we have a total of 2^{2N} different combinations. Each combination has $2N$ outcomes of atomic states—N outcomes each for evacuation of B_n and A_n. We compare the first N outcomes of any combination among the total of 2^{2N} with the last N outcomes of the same combination. When these are the same we get $j = 0$ and when they are reversed with each other we have $j = 1$. For example, if all the first N outcomes of a combination among 2^{2N} combinations are similar to the last N outcomes of the same combination then we have all j equal to 0. However, if any nth outcome of the first N outcomes is reversed with respect to the nth outcome of the last N then that $j_n = 1$. If all the outcomes of the first N are reversed with all the outcomes of the last N of that combination then we have all j equal to 1. This completes the procedure of measuring the Bell states $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1 \ldots A_N B_1 \ldots B_N)\rangle$. A determination of Bell state $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1 \ldots A_N B_1 \ldots B_N)\rangle$ projects the state of the field in cavities $C_1,C_2,\ldots,C_N$ into the entangled state $|\psi(C_1 \ldots C_N)\rangle$ as

$$|\psi(C_1 \ldots C_N)\rangle = \sum_{p_1,\ldots,p_N=0}^{1} C_{p_1,\ldots,p_N} \times \prod_{m=1}^{N} e^{i\pi j_m p_m} |(p_m+k_m)\mathrm{mod}2\rangle_{C_m}. \quad (35)$$

In the third and final step of the quantum teleportation, a manipulation of the cavities $C_1,C_2,\ldots,C_N$ needs to be done to bring state $|\psi(C_1 C_2 \ldots C_N)\rangle$ to form $|\psi(A_1 A_2 \ldots A_N)\rangle$. This transformation of state involves two steps. One is the removal of phases and the other is the appropriate transformation of photon numbers.

First we consider the transformation of phase only. It depends upon the value of j. If all j are 0, then we have to do nothing and the original state is recovered. However, if any j_n among N values of j is 1 then it has an additional phase with it. For the removal of this phase we send a two-level atom in a coherent superposition of states $|a\rangle$ and $|b\rangle$ through the cavity C_n in such a way that ground state $|b\rangle$ picks the phase. If the atom is detected in $|b\rangle$ then the original state is recovered, otherwise we have to repeat the process until it is detected in $|b\rangle$. If there are m values of j that are equal to 1 out of N values of j then we pass m atoms in coherent superposition of states $|a\rangle$ and $|b\rangle$ one by one from those m cavities and detect the atoms in ground state $|b\rangle$. If all the j are 1 then

we pass N atoms in $(|a\rangle+|b\rangle)/\sqrt{2}$ from all N cavities and detect atoms in ground state $|b\rangle$.

Next we consider the transformation of photon numbers in the cavities. This transformation depends upon the values of k. If all the k are 0 then we have to do nothing and the original state is recovered. However, if any k_n among N values of k is 1, then we have to change 0 and 1 photon from cavity C_n. For this purpose we pass a two level-atom in its ground state $|b\rangle$ through cavity C_n with a π pulse followed by its passage through a classical field again with a π pulse. Finally the atom passes through an empty cavity C_n' such that the atom in excited state $|a\rangle$ leaves the cavity in ground state $|b\rangle$ while leaving one photon inside the cavity and the atom in ground state $|b\rangle$ leaves the cavity in ground state with no photon inside the cavity. This leads the field states in the cavities $C_1, C_2, \ldots, C_N$ in the entangled state (31) and the teleportation is complete. If there are m values of k that are equal to 1 out of N values then we repeat the same process as above by sending m two-level atoms one by one in ground state $|b\rangle$ from each m cavity and proceed further as mentioned earlier until the completion of the process. If all the k are 1 then we pass N atoms in the ground state from all N cavities followed by a classical field that mixes $|a\rangle$ and $|b\rangle$ as $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |a\rangle$ and finally through N empty cavities. The field state in the cavities $C_1, C_2, \ldots, C_N$ have thus been projected to a state that has all the information about the amplitudes $C_{n_1, n_2, \ldots, n_N}$. This completes the transformation process and hence the teleportation of the N-qubit state.

IV. CONCLUSION

We have presented a scheme for the quantum teleportation of a two-qubit entangled state of the form (1) from a pair of cavities at the sender's end to another pair of cavities at the receiver's end. The scheme employs atomic interaction with high-Q cavities. We need two entangled states of two particles each for the teleportation of a two-particle entangled state. Sending one particle of each entangled state to the sender and the other particle to the receiver is sufficient to teleport the entangled state of two qubits. This scheme is then generalized for the teleportation of the N-qubit entangled state in N high-Q cavities of the form (31). For this purpose we need N entangled states of two qubits each. Sending one particle of each entangled state to the sending station and the other particle of that state to the receiving station is enough for the teleportation process.

The proposed scheme of teleportation consists of three steps. The first step involves preparation of quantum entangled states of type (2) and (3) between two high-Q cavities. The second and third steps involve optical Ramsey interferometry and single-photon transfer. All these require controlled interaction times between atoms and cavities, negligible cavity loss, and no spontaneous decay during the whole teleportation process. Controlling the interaction time in the cavities can easily be achieved by properly setting, through Stark field adjustment, the times during which atom is resonant with each cavity [6]. About the spontaneous decay we propose the Rydberg atom in circular states with principle quantum number ≈ 50. They have a long radiative lifetime (30 ms) and a very strong coupling to radiation [28]. A negligible cavity loss is also required during the whole process of teleportation. Cavity lifetimes for high-Q cavities should be long enough as all the interactions of atom with cavities should be completed before the cavity dissipation. High-quality factors of such cavities and control of atomic beams during the whole teleportation process may pose limitations on the suggested scheme.

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