Math (7)

REPORT OF THE PROJECT CU-MATH(7)

Project Title: STUDY OF MHD AND PLASMA PHYSICS.

* Magnetohydrodynamics

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Name & address of Reporting Institution: Physics Department, Quaid-i-Azam University, Islamabad.

Grant number:

Project number: CU - MATH (7).

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SUMMARY

(i) GENERAL REMARKS:

The aim of the project was to set up and develop the subject of MHD and Plasma Physics at the University of Islamabad.

In recent years the subject has assumed great importance due to many possibilities of its industrial and technological applications, in particular the possibility of achieving Controlled Thermonuclear Fusion leading to Fusion Reactors.

Unfortunately this subject was not even in existence in this University and for that matter anywhere in the country.

We have introduced this subject at M.Sc. and M.Phil. level. Indeed by now the subject has been fully incorporated into the teaching programme of the Physics Department. The outlines of the courses are separately attached.

By now more than 75 students have done their M.Sc. and M.Phil.* degrees with credit in Plasma Physics. The details are separately attached.

The PINSTECH Laser Group has started an M.Phil. training programme at the University for their new recruits. Such trainees are compulsorily required to offer the subject of Plasma Physics.

(ii) RESEARCH:

(a) NONLINEAR WAVES IN A TWO-COMPONENT HOT PLASMA.

Professor M.A.Rashid, Mr.Hafiz-ur-Rehman and myself studied nonlinear wave propagation in a hot, collisionless plasma consisting of electrons and ions. We assumed that the plasma was unbounded and that there was no ambient magnetic field. The model used was Boltzmann-Vlasov equation (B-V equation) in a Lorentz frame of reference S in which the space-dependence was eliminated.

We investigated transverse waves for the two cases: (i) the wave amplitude is small so that a perturbative expansion can be performed in terms of the amplitude. Truncating the series at an appropriate stage, a dispersion relation was obtained incorporating first-order non-linear correction. There was no restriction on the temperature in this case, (ii) assuming that the Plasma is not extremely hot so that the temperature effect can be treated as a small correction to the cold plasma case, we determined a dispersion relation describing a wave of finite amplitude.

In both cases the dispersion relations followed the pattern of a one-component electron plasma with ions forming a background of constant charge and current. We presented the results in a way that the electron and ion effects stood out separately.

This work forms a paper which has been accepted for publication in the Journal of Physics A. Vol.10, No.7 (1977).

(b) SOLITON AS A COHERENT STATE OF PHONONS.

Dr. K. Ahmed & myself studied the soliton-solution - the special solution of nonlinear dispersive equations in which non-linearity and dispersion balance each other so as to construct a constant profile solution. Such solutions seem to play an important role in many areas of physics including plasma physics.

In this regard we investigated a one-dimensional anharmonic lattice with N-particles equally spaced over a finite length. For such a model with cubic nonlinearity, it has been shown that the system satisfied a nonlinear differential equation(called KdV equation) which has a soliton solution. Such a solution is a Coherent state of phonons. We have tried to generalize this concept for an arbitrary degree of Nonlinearity.

This work has been written as Internal Report.

Continued.....

(c) Mr. Durrani and Dr. G.Murtaza studied the problem of Landau damping of transverse waves in the presence of a uniform magnetic field using Boltzmann-Vlasov equation and assuming small amplitude waves. The effect of the magnetic field was introduced through the expansion

where "f" is the distribution function. The resulting dispersion relation gives the Landau damping term incorporating correction due to the presence of the uniform magnetic field.

This work is in progress.

II DETAILED REPORT.

II.I. Non-linear Waves in a Two-component Hot Plasma.

II.2. Soliton As a Coherent State of Phonons.

To appear in J.Phy. A. Vol. 10, 7. (1977).

II. NON-LINEAR WAVES IN A TWO-COMPONENT HOT PLASMA

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1. INTRODUCTION

Following Clemmow[1,2], we study nonlinear wave propagation in a hot, collisionless plasma consisting of electr and ions. We assume that the plasma is unbounded and that there is no ambient magnetic field. The model used is the Boltzmann-Vlasov equation (B-V equations) in a Lorentz frame of referenc S in which the space-dependence is eliminated[3].

We investigate transverse waves for two cases, using perturbation technique: (i) When the plasma is not extremely hot so that the temperature effect can be treated as a small correction to the cold plasma case and the amplitude of the wave is large. (ii) When the wave amplitude is small and that there is no restriction on the temperature. In both cases, the dispersion relationsare obtained and the results are presented in a way that the electron and ion effects stand out separately

The plan of the paper is: Section 2 presents a genera formulation of the problem. Section 3 specialises to transvers propagation and develop the master equations (15) and (16). In sub-section 3.1 we record the results for the cold plasma. The main results of this paper are in sub-sections 3.2 and 3.3 describing the dispersion relations for strong waves (i.e. lar amplitude) with first order temperature effect and for weak wa (i.e. small amplitude) with first order non-linear correction keeping temperature arbitrary.

2. GENERAL FORMULATION

We consider S' as the laboratory frame in which the velocity of the wave is (0, 0, c/n) and S the frame in which there is no space-dependence and which is moving with velocity (0, 0, nc) relative to S' (n being the refractive index of the medium). All our calculations will be in frame S which can then be transformed to frame S' with the help of a Lorentz transformation.

Due to the absence of the spatial dependence of the fields in frame S, Maxwell's equations imply that the magnetic field $\underline{B}$ is constant and that the number densities of electrons and ions are equal, say N. Further, the curl of $\underline{B}$ equation is reduced to

$$-\epsilon_0 \dot{\underline{E}} = \sum_{\alpha=e,i} \underline{J}_\alpha \tag{1}$$

We consider the special case $\underline{B} = 0$. Then the relativistic B-V equations for electrons and ions will be

$$\frac{\partial f_\alpha}{\partial t} + \frac{q_\alpha}{m_\alpha c} \underline{E} \cdot \frac{\partial f_\alpha}{\partial \underline{u}_\alpha} = 0. \tag{2.}$$

Where $\underline{u}_e$ and $\underline{u}_i$ are the reduced velocities of electrons and ions respectively, defined in terms of the ordinary velocities $\underline{v}_e$ and $\underline{v}_i$ by

$$\underline{u}_\alpha = \frac{\gamma_\alpha \underline{v}_\alpha}{c}, \quad \gamma_\alpha = \left(1 + \underline{u}_\alpha^2\right)^{1/2}.$$

Also N $f_\alpha(\underline{u}_\alpha,t)$ is the distribution function. Now using $\underline{E} = -\dot{\underline{A}}$ and defining

$$\lambda_\alpha = -\frac{q_\alpha}{m_\alpha c} \qquad \text{where } q_i = +e \text{ and } q_e = -e$$

the B-V equation may be expressed as

$$\frac{\partial f_\alpha}{\partial t} + \lambda_\alpha \dot{\underline{A}} \cdot \frac{\partial f_\alpha}{\partial \underline{u}_\alpha} = 0. \tag{3}$$

These equations have a general solution

$$f_\alpha(\underline{u}_\alpha,t) = F_\alpha(\underline{u}_\alpha - \lambda_\alpha \underline{A}) \tag{4}$$

where F_α is an arbitrary function of its argument; $\underline{u}_\alpha - \lambda_\alpha \underline{A}$ is $\frac{1}{m_\alpha c}$ times the generalized momentum. Now

$$J_\alpha = N q_\alpha c \int \frac{\underline{u}_\alpha}{\gamma_\alpha} f_\alpha(\underline{u}_\alpha,t)\, d^3\underline{u}_\alpha$$

$$= \frac{N q_\alpha c}{\lambda_\alpha} \frac{\partial \mathbf{V}_\alpha}{\partial \underline{A}}$$

where

$$\mathbf{V}_\alpha = \int [1 + (\underline{u}_\alpha + \lambda_\alpha \underline{A})^2]^{\frac{1}{2}} F_\alpha(\underline{u}_\alpha)\, d^3\underline{u}_\alpha. \tag{5}$$

The equation (1) may therefore be rewritten as

$$\ddot{\underline{A}} + \sum_\alpha \frac{\omega_\alpha^2}{\lambda_\alpha^2} \frac{\partial V_\alpha}{\partial \underline{A}} = 0 \tag{6}$$

where

$$\omega_\alpha^2 = \frac{N q_\alpha^2}{\varepsilon_o m_\alpha}.$$

A is also a constant. With constant A, the equation (12) under appropriate initial conditions has a solution

$$\Phi = \omega t \;;\; \omega = \frac{h}{A^2} \qquad (13)$$

and the equation (10) reduces to the form

$$\frac{\partial V}{\partial A} = \frac{h^2}{A^3} \text{ or } \frac{1}{A}\frac{\partial V}{\partial A} = \omega^2. \qquad (14)$$

There is thus in S frame a monochromatic circularly polarized field of vector potential

$$\underline{A} = [A \cos(\omega t), A \sin(\omega t), A_z]$$

and

$$\underline{E} = A\omega[\sin(\omega t), -\cos(\omega t), 0]$$

where A, A_z and ω satisfy the equations

$$\sum_\alpha \frac{\omega_\alpha^2}{\lambda_\alpha^2} \frac{\partial V_\alpha}{\partial A_z} = 0 \qquad (15)$$

$$\sum_\alpha \frac{\omega_\alpha^2}{\lambda_\alpha^2} \frac{1}{A} \frac{\partial V_\alpha}{\partial A} = \omega^2. \qquad (16)$$

Transforming the results to the laboratory frame S´ again yields a purely transverse circularly polarized wave with velocity (0, 0, c/n) and angular frequency ω´. The fields in the laboratory frame S´ will be

$$\underline{E}' = E_0' \left\{ \sin[\omega'(t' - n z'/c)], -\cos[\omega'(t' - n z'/c)], 0 \right.$$

and

$$\underline{B}' = \frac{n}{c} \hat{\underline{z}} \times \underline{E}'$$

where the electric field amplitudes in S and S′ are related by

$$\frac{E_0}{\omega} = \frac{E_0'}{\omega'} = A. \qquad (17)$$

The dispersion relation is obtained by determining ω in terms of A from (15) and (16) and then substituting it in

$$\omega = (1 - n^2)^{\frac{1}{2}} \omega'. \qquad (18)$$

3.1 Dispersion Relation in Cold Plasma

The cold plasma results can be obtained by taking anisotropic streaming distributions, i.e.

$$F_\alpha(\underline{u}_\alpha) = \delta(\xi_\alpha)\, \delta(\eta_\alpha)\, \delta(\zeta_\alpha - u_{\alpha 0}) \qquad (19)$$

where

$$\underline{u}_\alpha = (\xi_\alpha, \eta_\alpha, \zeta_\alpha) = \left(\rho_\alpha \cos\phi_\alpha,\ \rho_\alpha \cos\phi_\alpha,\ \zeta_\alpha\right)$$

$$(20)$$

and $u_{\alpha o}$ is the reduced streaming velocity given by

$$u_{\alpha o} = \frac{v_{\alpha o}}{c}\left(1 - \frac{v_{\alpha o}^2}{c^2}\right)^{-\frac{1}{2}} = \frac{\gamma_{\alpha o}\, v_{\alpha o}}{c} . \qquad (21)$$

The velocities u_{eo} and u_{io} are related through the momentum conservation equation as

$$u_{io} + \mu\, u_{eo} = u_o \quad \text{(constant)},\ \mu = \frac{m_e}{m_i} \qquad (22)$$

The function V_α now takes a simpler form

$$V_\alpha = [1 + \lambda_\alpha^2 A^2 + (u_{\alpha o} + \lambda_\alpha A_z)^2]^{\frac{1}{2}} \equiv \Lambda_\alpha \quad \text{(say)} \qquad (23)$$

so that

$$\frac{1}{\lambda_\alpha}\frac{\partial V_\alpha}{\partial A_z} = \frac{L_\alpha}{\Lambda_\alpha} \quad ; \quad L_\alpha = u_{\alpha o} + \lambda_\alpha A_z \qquad (24)$$

and

$$\frac{1}{\lambda_\alpha^2 A}\frac{\partial V_\alpha}{\partial A} = \frac{1}{\Lambda_\alpha} . \qquad (25)$$

Now observing $\omega_i^2 = \mu\, \omega_e^2$ and $\lambda_i = -\mu\, \lambda_e$, and using equations (24) and (25), the equations (15) and (16) become

$$\frac{L_e}{\Lambda_e} - \frac{L_i}{\Lambda_i} = 0 \qquad (26)$$

and

$$\frac{1}{\Lambda_e} + \frac{\mu}{\Lambda_i} = \frac{\omega^2}{\omega_e^2} \qquad (27)$$

Also

$$L_\alpha = (1 + \lambda_\alpha^2 A^2)^{\frac{1}{2}} \Omega \tag{28}$$

where

$$\Omega = \frac{u_o}{(1 + \lambda_i^2 A^2)^{\frac{1}{2}} + \mu(1 + \lambda_e^2 A^2)^{\frac{1}{2}}} \tag{29}$$

Therefore

$$\begin{aligned} \Delta_\alpha &= (1 + \lambda_\alpha^2 A^2 + L_\alpha^2)^{\frac{1}{2}} \\ &= (1 + \lambda_\alpha^2 A^2)^{\frac{1}{2}} (1 + \Omega^2)^{\frac{1}{2}} \end{aligned} \tag{30}$$

Thus the dispersion relation (27) becomes

$$\frac{\omega^2}{\omega_e^2} = \frac{1}{(1 + \Omega^2)^{\frac{1}{2}}} \left[\frac{1}{(1 + \lambda_e^2 A^2)^{\frac{1}{2}}} + \frac{\mu}{(1 + \lambda_i^2 A^2)^{\frac{1}{2}}}\right] \tag{31}$$

Note that the ionic contribution which appears as an additive term can be significant, especially when the amplitude of the wave is large.

3.2 Dispersion Relation in Hot Plasma (First Order Temperature Correction)

Unless the plasma is extremely hot, we may use a perturbation technique to calculate first order temperature correction to the cold plasma result of the previous section. To do that, we first transform the cartesian variables of integration in the expression of V_α to the frame S''_α which is moving with velocity $(0,\ 0,\ v_{\alpha o})$ relative to S and then expand the integrand as a power series. The first order correction is obtained by truncating the series at the quadratic terms.

The Lorentz transformations are

$$\xi''_\alpha = \xi_\alpha\ ,\ \eta''_\alpha = \eta_\alpha\ ,\ \zeta''_\alpha = \gamma_{\alpha o}[\zeta_\alpha - \frac{v_{\alpha o}}{c}\gamma_\alpha]$$

where

$$\gamma_{\alpha o} = \left(1 - \frac{v^2_{\alpha o}}{c^2}\right)^{-\frac{1}{2}}$$

and

$$\gamma''_\alpha = \gamma_{\alpha o}\left(\gamma_\alpha - \frac{v_{\alpha o}}{c}\zeta_\alpha\right)$$

Also

$$d\zeta_\alpha = \frac{\gamma_\alpha}{\gamma''_\alpha}\, d\zeta''_\alpha$$

Therefore

$$d\xi''_\alpha\, d\eta''_\alpha\, d\zeta''_\alpha/\gamma''_\alpha = d\xi_\alpha\, d\eta_\alpha\, d\zeta_\alpha/\gamma_\alpha .$$

Further

$$N_{\alpha o}\, F_{\alpha o}(\xi''_\alpha\ , \eta''_\alpha\ , \zeta''_\alpha\ ) = N_\alpha\, F_\alpha(\xi_\alpha, \eta_\alpha, \zeta_\alpha)$$

where $N_{\alpha o}\ F_{\alpha o}$ is the equilibrium distribution function in S''_α. Also note that

$$N_\alpha = \gamma_{\alpha o}\, N_{\alpha o}$$

The expression for V_α then becomes

$$V_\alpha = \iiint_{-\infty}^{\infty} \Big\{1 + (\xi'' + \lambda_\alpha A_x)^2 + (\eta''_\alpha + \lambda_\alpha A_y)^2 +$$

$$[\gamma_{\alpha o}(\zeta''_\alpha + \frac{v_{\alpha o}}{c} \gamma''_\alpha) + \lambda_\alpha A_z]^2 \Big\}^{\frac{1}{2}} \quad x$$

$$\left(1 + \frac{v_{\alpha o}}{c} \frac{\zeta''_\alpha}{\gamma''_\alpha}\right) F_{\alpha o}(\xi''_\alpha, \eta''_\alpha, \zeta''_\alpha)\, d\xi''_\alpha\, d\eta''_\alpha\, d\zeta''_\alpha \qquad (32)$$

Now expanding the coefficient of $F_{\alpha o}$ in the integrand as a power series in ξ''_α, η''_α, ζ''_α and then performing integration term by term, assuming $F_{\alpha o}$ isotropic, we obtain

$$V_\alpha = \Lambda_\alpha + \frac{\theta_\alpha}{2\Lambda_\alpha}\left(1 + \gamma_{\alpha o}(\gamma_{\alpha o} + \frac{5v_{\alpha o}}{c} L_\alpha) + \frac{1 - u^2_{\alpha o} L^2_\alpha}{\Lambda^2_\alpha}\right) \qquad (33)$$

where

$$\theta_\alpha = \iiint_{-\infty}^{\infty} (\xi''^2_\alpha, \eta''^2_\alpha, \zeta''^2_\alpha)\, F_{\alpha o}\, d\xi''_\alpha\, d\eta''_\alpha\, d\zeta''_\alpha$$

$$\equiv \frac{K T_\alpha}{m_\alpha c^2} \qquad (34)$$

Note that we have truncated the series at the quadratic terms, ignoring higher order effects. Also Λ_α is the zero order term, which is the result of the cold plasma. Further, on differentiat the equation (33) we obtain

$$\frac{1}{\lambda^2_\alpha \Lambda} \frac{\partial V_\alpha}{\partial \Lambda} = \frac{1}{\Lambda_\alpha} - \frac{\theta_\alpha}{2\Lambda^3_\alpha}\left(1 + \gamma_{\alpha o}(\gamma_{\alpha o} + \frac{5v_{\alpha o}}{c} L\alpha) + \frac{3(1 - u^2_{\alpha o} L^2_\alpha)}{\Delta^2_\alpha}\right) \qquad (35)$$

$$\frac{1}{\lambda_\alpha}\frac{\partial V_\alpha}{\partial A_z} = \frac{L_\alpha}{\Lambda_\alpha} - \frac{\theta_\alpha}{2\Lambda_\alpha}\left(-5u_{\alpha o} + \frac{(3\gamma^2_{\alpha o} + 5u_{\alpha o} L_\alpha - 1)L_\alpha}{\Lambda^2_\alpha} + \frac{3(1 - u^2_{\alpha o} L^2_\alpha) L_\alpha}{\Lambda^4_\alpha}\right) \quad (36)$$

Since the analysis is correct only to the linear terms in θ_α, it is permissible to substitute for A_z in the co-efficient of θ_α in the equations (35) and (36) the expression given by cold plasma results i.e. equations (28) and (30). With these approximations, the above equations become

$$\frac{1}{\lambda^2_\alpha \Lambda}\frac{\partial V_\alpha}{\partial \Lambda} = \frac{1}{\Lambda_\alpha} - P_\alpha \theta_\alpha \quad (37)$$

$$\frac{1}{\lambda_\alpha}\frac{\partial V_\alpha}{\partial \Lambda_z} = \frac{L_\alpha}{\Lambda_\alpha} - Q_\alpha \theta_\alpha \quad (38)$$

where

$$P_\alpha = \frac{1}{2(1 + \lambda^2_\alpha \Lambda^2)(1 + \Omega^2)^{3/2}}\left(5u_{\alpha o}\Omega + \frac{(1+\gamma^2_{\alpha o})(1+\Omega^2) - 3u^2_{\alpha o}\Omega^2}{(1 + \Omega^2)(1 + \lambda^2_\alpha A^2)^{1/2}} + \frac{3}{(1 + \Omega^2)(1 + \lambda^2_\alpha \Lambda^2)^{3/2}}\right) \quad (39)$$

$$Q_\alpha = \frac{1}{2(1+\lambda^2_\alpha A^2)^{1/2}(1+\Omega^2)^{3/2}}\left(-5u_{\alpha o} + \frac{[(3\gamma^2_{\alpha o} - 1)(1+\Omega^2) - 3u^2_{\alpha o}\Omega^2]\Omega}{(1+\Omega^2)(1 + \lambda^2_\alpha A^2)^{1/2}} + \frac{3\Omega}{(1+\Omega^2)(1+\lambda^2_\alpha A^2)^{3/2}}\right)$$

(40)

Now substituting the above equations in (15) and (16) we obtain

$$\frac{L_e}{\Lambda_e} = \frac{L_i}{\Lambda_i} + (Q_e\,\theta_e - Q_i\,\theta_i) \qquad (41)$$

and

$$\frac{1}{\Lambda_e} + \frac{\mu}{\Lambda_i} - (P_e\,\theta_e + \mu\,P_i\,\theta_i) = \frac{\omega^2}{\omega_e^2}\,. \qquad (42)$$

The next step is to eliminate Λ_z so as to obtain ω in terms of the amplitude of the wave A only. In the circumstance that the waves are large amplitude, this is achieved by squaring (41), using (33) and continuing to work only to the linear terms in θ_α. After some algebra we obtain

$$\frac{1}{\Lambda_e} + \frac{\mu}{\Lambda_i} = - \frac{L_i}{\left[(1 + \lambda_e^2 A^2)(1 + \lambda_i^2 A^2)\right]^{1/2}}(Q_e\,\theta_e - Q_i\,\theta_i)$$

where we have assumed $A^2 >> 1$ i.e., the waves are strong waves. Using the cold plasma expression for L_i we get

$$\frac{1}{\Lambda_e} + \frac{\mu}{\Lambda_i} = (1 + \lambda_i^2 A^2)^{-1/2}\,\Omega\,(Q_i\,\theta_i - Q_e\,\theta_e) \qquad (43)$$

so that

$$\frac{\omega^2}{\omega_e^2} = \left(\frac{\Omega\,Q_i}{(1 + \lambda_e^2 A^2)^{1/2}} - \mu\,P_i\right)\theta_i - \left(\frac{\Omega\,Q_e}{(1 + \lambda_e^2 A^2)^{1/2}} + P_e\right)\theta_e \qquad (44)$$

The dispersion relation in S´ is obtained by using (18)

$$n^2 = 1 - \Gamma \frac{\omega_e'^2}{\omega'^2} \left\{ \left(\frac{9Q_i}{(1 + \lambda_e^2 \Lambda^2)^{\frac{1}{2}}} - \mu P_i \right) \theta_i - \left(\frac{9Q_e}{(1 + \lambda_e^2 \Lambda^2)^{\frac{1}{2}}} + P_e \right) \theta_e \right\} \qquad (45)$$

Where $\Gamma = (1-n^2)^{-1/2}$

From the expression of Q_α and P_α, it is evident that Q_i and P_i can be large compared to Q_e and P_e respectively. We may therefore conclude that the ionic contributions can be significant unless the ion-temperature is negligibly small.

3.3 Dispersion Relation for Weak Waves With First Order Non-linear Correction

In this section we treat the amplitude of the wave as a small parameter, and then use the perturbation technique to determine the dispersion relation incorporating first order non-linear correction, but the temperature in this case is unrestricted. To be explicit, we shall expand V_α in powers of Λ and Λ_z and then truncate the series at terms of order Λ^3. With this V_α, we calculate its differentials $\frac{\partial V_\alpha}{\partial \Lambda}$ and $\frac{\partial V_\alpha}{\partial \Lambda_z}$ and then substitute them in equations (15) and (16). That will yield the desired dispersion relation.

For convenience we use the cylindrical polar coordinate $\underline{u}_\alpha = (\rho_\alpha \cos \phi_\alpha, \rho_\alpha \sin \phi_\alpha, \zeta_\alpha)$ and adopt the notation

$$\langle P(\rho_\alpha, \phi_\alpha, \zeta_\alpha) \rangle = \int P(\rho_\alpha, \phi_\alpha, \zeta_\alpha) \, F_\alpha(\underline{u}_\alpha) \, d^3\underline{u}_\alpha$$

$$\frac{1}{D_\alpha} = \frac{1}{\gamma_\alpha}\Big(1 - \frac{1}{\gamma_\alpha^2}\lambda_\alpha A \rho_\alpha \cos\phi_\alpha - \frac{1}{2\gamma_\alpha^2}(\lambda_\alpha^2 A^2 + 2\lambda_\alpha \Lambda_z \zeta_\alpha)$$

$$+ \frac{3}{2\gamma_\alpha^4}\lambda_\alpha^2 A^2 \rho_\alpha^2 \cos^2\phi_\alpha + \frac{3}{2\gamma_\alpha^4}\lambda_\alpha A \rho_\alpha \cos\phi_\alpha (2\lambda_\alpha \Lambda_z \zeta_\alpha$$

$$+ \lambda_\alpha^2 A^2) - \frac{5}{2\gamma_\alpha^6}\lambda_\alpha^3 A^3 \rho_\alpha^3 \cos^3\phi_\alpha\Big) \qquad (49)$$

where we have assumed that Λ_z is of the order of A^2. This assumption is indeed true for cold plasma, as may be seen from the equation (28) and is verified a pasteriori for the hot plasma (see equation 51).

Hence, to the order A^2

$$\frac{1}{\lambda_\alpha}\frac{\partial V_\alpha}{\partial \Lambda_z} = \left\langle \frac{\lambda_\alpha \Lambda_z}{\gamma_\alpha} + \frac{\zeta}{\gamma_\alpha}\left(1 - \frac{\lambda_\alpha^2 A^2 + 2\lambda_\alpha \Lambda_z \zeta_\alpha}{2\gamma_\alpha^2}\right.\right.$$

$$\left.\left. + \frac{3\lambda_\alpha^2 A^2 \rho_\alpha^2}{4\gamma_\alpha^4}\right)\right\rangle \qquad (50)$$

Here we observe that

$$\left\langle \frac{\zeta_i}{\gamma_i} \right\rangle = \left\langle \frac{\zeta_e}{\gamma_e} \right\rangle$$

because from the equation (1), it is clear that

$$J_e + J_i \Big|_{\text{at } A=0} = 0$$

i.e.

$$\int \frac{\underline{u}_e}{\gamma_e} F_e(\underline{u}_e)\, d^3\underline{u}_e = \int \frac{\underline{u}_i}{\gamma_i} F_i(\underline{u}_i)\, d^3\underline{u}_i$$

Therefore the relation (15) determines A_z as

$$\lambda_e A_z = \frac{\left\langle \frac{\zeta_e}{2\gamma_e^3}\left(1 - \frac{3\rho_e^2}{2\gamma_e^2}\right) - \frac{\mu^2 \zeta_i}{2\gamma_i^3}\left(1 - \frac{3\rho_i^2}{2\gamma_i^2}\right) \right\rangle}{\left\langle \frac{1+\rho_e^2}{\gamma_e^3} + \mu \frac{1+\rho_i^2}{\gamma_i^3} \right\rangle} \lambda_e^2 A^2 \qquad (51)$$

Also to the order A^2, it is found that

$$\frac{1}{\lambda_\alpha^2 A} \frac{\partial V}{\partial A} = \left\langle \frac{1}{\gamma_\alpha} - \frac{\rho_\alpha^2}{2\gamma_\alpha^3} - \frac{\zeta_\alpha}{\gamma_\alpha^3}\left(1 - \frac{3\rho_\alpha^2}{2\gamma_\alpha^2}\right) \lambda_\alpha A_z - \frac{1}{2\gamma_\alpha^3}\left(1 - \frac{3\rho_\alpha^2}{\gamma_\alpha^2} + \frac{15}{8}\frac{\rho_\alpha^4}{\gamma_\alpha^4}\right) \lambda_\alpha^2 A^2 \right\rangle \qquad (52)$$

Now using (51) and (52) in equation (16) we get

$$\frac{\omega^2}{\omega_0^2} = X_0 - \frac{1}{2} X_1 \lambda_e^2 A^2 \qquad (53)$$

where

$$X_0 = \left\langle \left(\frac{1}{\gamma_e} - \frac{\rho_e^2}{2\gamma_e^3}\right) + \mu \left(\frac{1}{\gamma_i} - \frac{\rho_i^2}{2\gamma_i^3}\right) \right\rangle \qquad (54)$$

where

$$X_1 = \frac{\left\langle \frac{\zeta_e}{\gamma_e^3}\left(1 - \frac{3\rho_e^2}{2\gamma_e^2}\right) - \frac{\mu^2 \zeta_i}{\gamma_i^3}\left(1 - \frac{3\rho_i^2}{2\gamma_i^2}\right) \right\rangle^2}{\left\langle \frac{1 + \rho_e^2}{\gamma_e^3} + \mu \frac{1 + \rho_i^2}{\gamma_i^3} \right\rangle} +$$

$$\left\langle \frac{1}{\gamma_e^3}\left(1 - \frac{3\rho_e^2}{\gamma_e^2} + \frac{15}{8}\frac{\rho_e^4}{\gamma_e^4}\right) + \frac{\mu^3}{\gamma_i^3}\left(1 - \frac{3\rho_i^2}{\gamma_i^2} + \frac{15}{8}\frac{\rho_i^4}{\gamma_i^4}\right) \right\rangle$$

(55)

Note that the effects of the ionic motion stand out in the co-efficient of μ and taking $\mu = 0$ gives the old results of the one-component plasma.

In the frame S′ the dispersion relation takes the form:

$$n^2 = 1 - \Gamma\left(X_0 - \frac{1}{2} X_1 \lambda_e^2 A^2\right)\left(\frac{\omega'}{\omega}\right)^2 . \qquad (56).$$

REFERENCES

(1) CLEMMOW, P.C., 1974, J. Plasma Physics, 12, 297.

(2) CLEMMOW, P.C., 1975, J. Plasma Physics, 13, 231.

(3) WINKLES, B.B. and ELDRIDGE, O., 1972, Phys. Fluids, 15, 1790.

CONCLUSION AND FUTURE EXTENSION.

The development of Plasma Physics began with linear effects and at first it seemed that the objective of, for example, the theory of plasma confinement was to look for those ranges of physical parameters for which the plasma would be stable. Actually, in the range of stability the fields in the plasma do not grow, and the application of the linear approximation is justified. However, research eventually disclosed, at first by theory and subsequently by experiment, a continual chain of new plasma instabilities. As a result researchers began to realize that a plasma was highly prone to become unstable and that the presence of instabilities was its most characteristic attribute as a state of matter. It soon became clear that the nonlinear effects were the most important factor in comprehending the physical processes in a plasma.

Very recently there has been a considerable advance in the study of non-linear effects, but many of the problems touched on are still far from the solution, and in many cases we have only special solutions.

In 1972, Winkles and Eldridge tackled this problem from a new aspect and introduced a technique which can simplify the problem. Using this technique - called the space independent technique - Clemmow worked on non-linear waves in cold plasma(1974) and then on non-linear waves in hot electron plasma (1975), assuming the absence of ambient magnetic field. We also used the same technique and worked for two-component hot plasma, assuming the absence of externally applied ambient magnetic field, and got the dispersion relations for different cases.

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This work needs to be extended further so as to investigate the effects on the dispersion relation of the presence of the ambient magnetic field. Such an investigation will be more in line with the real physical situations.

So far in our work we have confined ourselves to the study of the transverse waves. There is therefore, an obvious need to extend this work to longitudinal waves so as to complete the solution of the problem.

II.2. SOLITON AS A COHERENT STATE OF PHONONS.

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(INTERNAL REPORT).

Recently Ichikawa et al[1] studied the old 'recurrence problem' of Fermi, Pasta and Ulam of a one-dimensional anharmonic lattice and gave an explanation emphasizing the discrete character of the system in terms of phonons, in contrast to the Zabusky's Continuum Model.[2] In their work, the Korteweg-de Vries (KdV) equation is derived on the basis of a coherent state representation for the interacting phonons, and it is explicitly shown that a soliton solution can be given a quantum mechanical interpretation as a coherent state of excited phonons in the system. In the present note, we extend this work to a generalized one-dimensional lattice with an arbitrary degree of anharmonicity n and obtain a generalized KdV equation that describes the system. The quantum content of the one-soliton state is maintained as before. An expression for the effective mass of the soliton is also given in terms of the degree n and the coupling g_n of non-linearity. In this work, we shall mostly follow the notation of reference (1).

We consider a one-dimensional generalized anharmonic lattice with N-particles equally spaced over a length $L = N\ell$ described by the Hamiltonian

$$H = \sum_{r=1}^{N} \frac{1}{2}\left[m\dot{y}_r^2 + K(y_{r+1}-y_r)^2 + \frac{1}{n}Kg_n(y_{r+1}-y_r)^n\right], \quad (n = 3,4,5,\dots) \qquad (1)$$

where y_r, $\dot{y}_r$ are the displacement and velocity of the rth particle with mass m, K is the linear spring constant and $g_n > 0$ measures the strength of the non-linearity. Introducing the normal mode expansions

$$y_r = \frac{1}{\sqrt{N}} \sum_k \sqrt{\frac{\hbar}{2m\omega(k)}} \, (a^*_{-k} + a_k) \, e^{ikx_r}$$

$$\dot{y}_r = \frac{i}{\sqrt{N}} \sum_k \sqrt{\frac{\hbar\,\omega(k)}{2m}} \, (a^*_{-k} - a_k) \, e^{ikx_r} \qquad (2)$$

where $x_r = r\ell$ gives the position of the rth particle. Now quantizing the system in the usual way, we obtain

$$H = H_0 + H'$$

$$H_0 = \sum_k \hbar\omega(k) \, (a^*_k a_k + 1/2)$$

$$H' = \sum_{k_1, k_2, \dots, k_n} \Delta(k_1 + k_2 + \dots + k_n) \, \varphi(k_1, k_2, \dots, k_n) \times$$

$$\times (a^*_{-k_1} + a_{k_1})(a^*_{-k_2} + a_{k_2}) \dots (a^*_{-k_n} + a_{k_n}), \qquad (3)$$

where

$$\Delta(k) = \frac{1}{N} \sum_{r=1}^{N} e^{irlk} \qquad (4)$$

$$\omega^2(k) = 4 \frac{K}{m} \sin^2\left(\frac{\ell k}{2}\right) \qquad (5)$$

$$\varphi(k_1, k_2, \dots, k_n) = \frac{1}{2}\left(\frac{1}{n} K g_n\right)\left(\frac{\hbar}{2m}\right)^{n/2} \frac{(2i)^n}{(\sqrt{N})^{n-2}} \exp\left\{-\frac{i\ell}{2}(k_1 + \dots + k_n)\right\} \times$$

$$\times \left\{\omega(k_1)\omega(k_2) \dots \omega(k_n)\right\}^{-1/2} \sin\frac{\ell k_1}{2} \sin\frac{\ell k_2}{2} \dots \sin\frac{\ell k_n}{2}. \qquad (6)$$

And

$$[a_k, a^*_{k'}] = \Delta(k - k') \; ; \; [a_k, a_{k'}] = 0 = [a^*_k, a^*_{k'}]$$

Now we introduce, following Glauber [3], the coherent state of phonons $|\alpha_k\rangle$ defined as:

$$a_k |\alpha_k\rangle = \alpha_k |\alpha_k\rangle$$

$$|\alpha_k\rangle = \exp\left(-\tfrac{1}{2}|\alpha_k|^2\right) \sum_{n_k=0}^{\infty} \frac{(\alpha_k)^{n_k}}{\sqrt{n_k!}} |n_k\rangle$$

with average occupation number given by a Poisson distribution with mean value $\langle n_k\rangle = |\alpha_k|^2$. Then the expectation value of the displacement is given by

$$\langle \alpha_k | y_r | \alpha_k \rangle = \frac{1}{\sqrt{N}} \sum_k y(k) e^{ikx_r}$$

where

$$y(k) = \sqrt{\frac{\hbar}{2m\omega(k)}}\, (\alpha^*_{-k} + \alpha_k) .$$

Using the temporal evolution of the expectation values of the Heisenberg creation and destruction operators with respect to a coherent state, we obtain the equation of motion for the kth mode displacement $y(k)$:

$$\ddot{y}(k) = -\omega^2(k)\, y(k) - 2n \sqrt{\frac{\omega(k)}{2m\hbar}} \cdot \left(\frac{2m}{\hbar}\right)^{\frac{n-1}{2}} \cdot \sum_{k_1,\dots,k_n} \phi(k_1,\dots,k_n) \times$$

$$\times\, \Delta(k_1+\dots+k_n)\, \Delta(k+k_1) \sqrt{\omega(k_1)\dots\omega(k_n)}\; y(k_2)\dots y(k_n) . \qquad (7)$$

If we neglect the contributions from the large wave-number phonons, we may approximate ϕ and ω (eqns. (5) and (6)) as

$$\phi(k_1,\cdots,k_n) = \frac{1}{2}\left(\frac{1}{n}Kg_n\right)\left(\frac{\hbar}{2m}\right)^{n/2}\cdot\frac{(2i)^n}{(\sqrt{N})^{n-2}}\cdot\frac{lk_1}{2}\cdot\frac{lk_2}{2}\cdots\frac{lk_n}{2}\times$$

$$\times\left\{\omega(k_1)\cdot\omega(k_2)\cdots\omega(k_n)\right\}^{-1/2}$$

$$\omega(k) \cong \sqrt{\frac{K}{m}}\, l|k|\left(1-\frac{1}{24}l^2k^2\right)$$

$$= s|k|\left(1-\frac{1}{24}l^2k^2\right), \text{ where } s=\sqrt{\frac{K}{m}}\,l$$

that the equation of motion (7) becomes

$$\ddot{y}(k)+s^2k^2\left(1-\frac{1}{12}l^2k^2\right)y(k) = \frac{1}{8}Ks^2\left(\frac{l}{2\sqrt{N}}\right)^{n-2}(2i)^n\cdot k\times$$

$$\times\sum_{k_2,\dots,k_n}\Delta(-k+k_2+\cdots+k_n)k_2k_3\cdots k_n\,y(k_2)\cdots y(k_n) \qquad (8)$$

w defining a new variable u(k,t) and its Fourier transform

$$u(k,t) = ik\,y(k,t)$$

$$u(x,t) = \frac{1}{\sqrt{N}}\sum_k u(k,t)\,e^{ikx},$$

may Fourier transform Eqn.(8) into a non-linear differential uation which governs the dynamics of our generalized anharmonic ttice:

$$\frac{\partial^2}{\partial t^2}u(x,t) - s^2\frac{\partial^2}{\partial x^2}u(x,t) - \frac{1}{12}s^2l^2\frac{\partial^4}{\partial x^4}u(x,t) - g_n\frac{s^2l^{n-2}}{2}\times$$

$$\times\frac{\partial^2}{\partial x^2}\left(u(x,t)\right)^{n-1} = 0. \qquad (9)$$

is equation is a generalization of the Boussinesq equation and n be converted to the KdV type by using the reductive perturbation method[4] with the following expansion and space-time rescaling

$$u = \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \cdots$$

$$\xi = \epsilon^{\left(\frac{n-2}{2}\right)}(x-t)$$

$$\tau = \epsilon^{3\left(\frac{n-2}{2}\right)} t ,$$

into the form

$$\frac{\partial}{\partial\xi}\left[\frac{\partial}{\partial\tau} u^{(1)} + \frac{1}{24} s^2 \ell^2 \frac{\partial^3}{\partial\xi^3} u^{(1)} + g_n \frac{s^2 \ell^{n-2}}{4} \frac{\partial}{\partial\xi}\left(u^{(1)}\right)^{n-1}\right] = 0 \tag{10}$$

- the generalized KdV equation. Notice that on choosing n = 3,4 one recovers the standard KdV equation and its modified form for cubic and quartic non-linearities, respectively. Returning to the original variables, the above equation (10), becomes

$$\frac{\partial}{\partial t} u(x,t) + \frac{\partial}{\partial x} u(x,t) + \frac{1}{24} s^2 \ell^2 \frac{\partial^3}{\partial x^3} u(x,t) + g_n \frac{s^2 \ell^{n-2}}{2} \frac{\partial}{\partial x}\left[u(x,t)\right]^{n-1} = 0 \tag{11}$$

The generalized KdV equation admits one soliton solution which is given by[3]

$$u(x,t) = \eta \left[\operatorname{sech} a_n (x - b_n s t)\right]^{\frac{2}{n-2}} \tag{12}$$

where

$$a_n^2 = (n-2)^2 \left(\frac{3}{n} g_n \eta^{n-2} \ell^{n-4}\right)$$

$$b_n = \left(1 + \frac{1}{2n} g_n \eta^{n-2} \ell^{n-2}\right).$$

It is easy to see that n = 3 reproduces the results of Ichikawa et al[1].

Finally, it is straig tforward to verify that the one-soliton state given by Equation (12) is now a coherent state of excited phonons with amplitude α_k as

$$\alpha_k = \frac{-i}{\sqrt{N}} \sqrt{\frac{m\omega(k)}{2\hbar}} \left(1 + b_n \frac{sk}{\omega(k)}\right) 4^{1/n-2} \left(\frac{\eta}{2a_n \ell k}\right) \times$$

$$\times\, e^{-ikb_n st} B\left(\frac{1}{n-2} + \frac{ik}{2a_n}; \frac{1}{n-2} - \frac{ik}{2a_n}\right)$$

(13)

and with average occupation number of phonons given by $\langle n_k \rangle = |\alpha_k|^2$. Here the beta function $B(\mu, \nu)$ enters through the Fourier transform of the one-soliton solution:

$$u(k,t) = \frac{\sqrt{N}}{L} \int_{-\infty}^{\infty} u(x,t)\, e^{-ikx}\, dx$$

$$= \frac{4^{1/n-2}}{\sqrt{N}\, 2a_n \ell}\, e^{-ikb_n st}\, B\left(\frac{1}{n-2} + \frac{ik}{2a_n}, \frac{1}{n-2} - \frac{ik}{2a_n}\right)$$

or cubic and quartic non-linearities, the beta function simplifies $\frac{\pi k}{2a_3}$ cosech $(\pi k/2a_3)$ and π sech $(\pi k/2a_4)$, respectively. We y also estimate mass associated with a soliton from the momentum uation

$$\underset{\sim}{P} = \sum_k \hbar \underset{\sim}{k} \langle n_k \rangle$$

$$= m_n^* \left(1 + \frac{\alpha_n}{2n} \eta^{n-2} \ell^{n-2}\right) \underset{\sim}{s} \qquad (14)$$

where m_n^* defined as the effective mass of the soliton is given by

$$m_n^* = \frac{1}{4\sqrt{\pi}} \frac{\eta^2}{a_n l} 4^{\frac{2}{n-2}} \frac{\Gamma^2\left(\frac{1}{n-2}\right)\Gamma^2\left(\frac{1}{n-2}+\frac{1}{2}\right)}{\Gamma\left(\frac{2}{n-2}\right)\Gamma\left(\frac{2}{n-2}+\frac{1}{2}\right)} m. \quad (15)$$

We thank Dr. S.M.M.R.Choudhry for useful discussions. We also wish to thank Professor Abdus Salam, the International Atomic Energy and UNESCO for hospitality at the ICTP, Trieste where this work was done.

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(1) Yoshi H. Ichikawa, Novuo Yajima and Kaoru Takano, Prog. Th. Phys., 55, 1783 (1976). See other references in this paper.

(2) See reference quoted in (1).

(3) R.J. Glauber, Phys. Rev. 131,2766 (1963).

(4) T. Taniuti, Prog. Th. Phys. Suppl. No. 55 (1974).

LIST OF PUBLICATIONS.

(1) Nonlinear waves in a Two - component hot plasma:
To appear in J.Phys.A. Vol. 10, No.7 (1977).

(2) Soliton as a Coherent State of Phonons. (Internal Report).

LIST OF STUDENTS WHO STUDIED PLASMA PHYSICS DURING M.Sc. & M.Phil.

Year	Semester	Exam. held in.	#students in M.Sc.	#students in M.Phil I	# students in M.Phil II
1974	Fall	Jan. 1975	8	Nil	Nil
1975	Spring	June 1975	12	9	Nil
1975	Fall	Jan., 1976	10	Nil	6
1976	Spring	June 1976	10	6	Nil
1976	Fall	Jan., 1977	9	Nil	4
1977	Spring	Not yet held	6	8	Nil
		TOTAL	58	23	10

TOTAL No. OF STUDENTS: 91

One Ph.D. student has also been registered this year.

COURSES OF PLASMA PHYSICS FOR M.Sc. & M.Phil.

P-403 Plasma Physics for M.Sc. Fourth Semester.

Charged Particles in Electromagnetic Fields, Motion in Time Varying Magnetic Field and Crossed Electric and Magnetic Fields; Radiation from Accelerated Charges; Electrical Neutrality of Plasma; Plasma Oscillation; Plasma Confinement; Fluid Description of Plasma; Plasma Applications.

Books Recommended:

1. Tanenbaum, B.S. Plasma Physics. McGraw-Hill.(1967)
2. Thomson, W.B. Int. to Plasma Physics. Addison-Wesley,(1964).
3. Spitzer, L. Physics of Fully Ionised Gases. John-Wiley,(1962)
4. Glasstone, S. Controlled Thermo-Nuclear Reaction. Nostrand (1960)

P-613 Plasma Physics-I for M.Phil. 1st Semester:

The Vlasov theory for plasma waves; Landau Damping; The two-stream Instability; the Debye potential Problems and Plasma Sheaths. The Boltzmann Equation for a Plasma; the H-Theorem; Transport Phenomena; Various models to solve the Boltzmann equation; the BBGKY Equations for a Plasma; Non-linear Vlasov theory of Plasma Waves & Instabilities.

Books Recommended:

1. Plasma Physics, B. Samuel Tanenbaum, McGraw-Hill, Book Company [illegible]
2. Principles of Plasma Physics N.A.Krall and A.W.Trivelpiece. McGraw-Hill, Book Company (1973).

continued.......

P-713 **<u>Plasma Physics-II for M.Phil 2nd Semester</u>**

General features of Hydrodynamics, Equation of State Hydrodynamical shocks in various Geometries, their Reflection. Shock Sequence, Compression in a Single convergent shock. Kinetic and Thermal Energy in the Shock. Rayleigh-Taylor Instability. BBGKY Kinetic Theory of Plasma; Vlasov theory of Plasma Stability; Non-linear Vlasov theory of Plasma waves & instabilities. Plasma Diagnostic techniques; Plasma Spectroscopy; controlled thermo-nuclear Fusion & Fusion Devices; Laser Produced Plasmas and laser Fusion.

<u>Books Recommended:</u>

1. High Pressure Physics & Chemistry Bradley.

2. Principles of Plasma Physics, by Krall N.A. & Trivelpiece, A.W. McGraw-Hill, (1973)

3. Plasma Physics by Tanenbaum, B.S. McGraw-Hill,(1967).
4. Plasma Diagnostics by W.Lotch-Holtgraven(1968).
5. Plasma Spectroscopy by H.R.Griem; McGraw-Hill(1964)

6. Plasma and Controlled Fusion by D.J. Rose and M.Clark Jr.MIT - Press, (1961).

LIST OF SCIENTISTS WHO PARTICIPATED IN THE RESEARCH WORK OF THE PROJECT.

1. Prof. M.A. Rashid, Principal Investigator.
   (Left for Nigeria in Jan. 1975).

2. Prof. G. Murtaza, Principal Investigator,
   (Since Jan. 1975).

3. Mr. Wasim Ahmad Roshan, Research Officer.
   (Left for Ph.D. programme in U.S.A. August 1975).

4. Mr. Hafeez-ur-Rehman, Research Officer
   (Since April, 1976).

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