FINAL RESEARCH REPORT

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7.	Signature of the Principal Investigator:	M. Saleem
8.	Signature of the Institutional Head:	DEAN Faculty of Science University of the Punjab LAHORE.

Summary:

During the period 1-8-76 to 31-7-79, we studied theoretical models as well as experimental techniques for "inclusive reactions" in various proceedings and review articles. The special emphasis has been on the reactions $p+p \rightarrow p+X$ and $p+d \rightarrow d+X$. It is hoped that study of the current literature on this topic will enable us to propose a model for such reactions. During this period, some theoretical models for inclusive reactions have been proposed. We noticed that for initiating the work on a specific research problem, it was necessary to examine the experimental techniques as well. The proceedings, review articles and books have been studied with the aforementioned reactions in view.

Detailed Report

This research project was started in August 1976. At that time our field of interest was Exclusive Reactions. When we started scanning the literature on Inclusive Reactions, it was realised that enormous work had been done in this field which was expanding at a very rapid rate. During the first year we concentrated on Gribov's reggeon calculus and study of literature on inclusive reaction. During the second year we continued to survey the literature as described below. We have, inter alia, studied the following review articles.

1. Phenomenology of Inclusive reactions by E.L.Berger.

2. Inclusive processes at High Energy by V.Exhela et al.

3. Lectures on Inclusive Reactions by R.C.Arnold.

4. Particle Production in Hadron Physics by S.Humble.

5 Regge Phenomenology of Inclusive Reactions by Chan.

6. Reggeon Calculus by Baker.

7. Introduction to Particle Production in Hadron Physics By S.Humble.

We studied in detail the dynamics underlying inclusive reactions as well as the experimental techniques. Remembering that the single particle distribution is a function of three variables, say, s, q, r, we analysed the variation of distributions with s. We also considered the variation of distributions with q and r for fixed s. Benecke, Chou, Yang and Yen consider either the Laboratory or projectile rest frame. For example, in the laboratory frame, their limiting fragmentation hypothesis is that $f_c(s, q^L, r)$ approaches an asymptotic limit for large s, i.e,

$$f_c(s, q^L, r) \longrightarrow \hat{f}_c(q^L, r) \qquad (1)$$

provided q^L is held fixed as $s \to \infty$. Since initially the target particle b is at rest in this frame, while the beam particle (projectile a) has a momentum increasing with s, they consider such detected particles c with finite momentum q^L as representing fragments of the target particle. That is to say, $\hat{f}_c \quad (q^L \quad , r)$ reflects the break-up of the target particle b and is independent of the projectile except in determining an overall normalization factor. A handy notation for this process is given by

$$b \xrightarrow{a} c$$

which means the fragmentation of b into a particle c under the impact of particle a. A similar statement to (1) holds in the projectile rest frame, i e. for the process $a \xrightarrow{b} c$

$$f_c(s, q^P, r) \rightarrow \tilde{f}_c(q^P, r) \qquad (2)$$

when q^P is held fixed as $s \rightarrow \infty$. The extension of this limiting fragmentation hypothesis to the case of two - and more-particle distributions is immediate.

The intuitive argument for the limiting fragmentation hypothesis is based on the geometrical (droplet) picture for diffractive scattering considered by Wu and Yang, Byers and Yang and Chou and Yang. In this model the two colliding particles are interpreted as two spatially extended objects. As the energy increases, the projectile undergoes increasing Lorentz contraction as seen by the target, i e. in the Lab. frame. However, it is also observed that in the case of elastic scattering $d\sigma_{el}/dt$ and therefore also σ_{el} as well as σ_{tot} apparently all approach asymptotically constant values. Hence, whatever the dynamical mechanism which controls and transfers momentum and quantum numbers between the matter in the projectile and the matter in the target, it does not seem to change appreciably as the energy increases and the projectile is still further contracted. In the same way, it is argued, one might expect the excitation and break-up of the target also to be asymptotically independent of the energy, and therefore that the single- and many-particle distributions approach limiting values.

During the last few years ISR and FNAL have been producing very interesting results from which there has been considerable progress in understanding the high-energy properties and the underlying regularities of diffraction mechanisms. A number of experiments are **being performed to investigate in detail the mass and** four-momentum transfer dependence of the inclusive cross-sections as well as the production and decay properties of specific final states throughout the ISR energy range. A rather substantial amount of data on double diffractive processes, which were unambiguously identified and measured, is providing us wit new tools to study the dynamical properties shared by different classes of diffractive reactions. Within the framework of multiparticle reactions diffraction diassociation is known to play a dominant role in the high-energy behaviour of n-body collisions, with n small compared to the average value at each energy. Several theoretical expectations deal with exchange of vacuum quantum numbers, factorization relations among various reactions, spin-parity selection rule, helicity conservation in appropriate reference frames, and so on. To consider fundamental questions which might be relevant to the over-all picture of the new, high-energy results, we note the following points:

a) The s-dependence of the diffractive component in exclusive reactions; in particular, the energy dependence of the production cross-section for isobar states.

b) The mass dependence of the inclusive diffractive cross-section for large produced masses.

c) The resonance composition of final states. In particular, do invariant mass distributions reach a limiting behaviour with increasing s as inclusive spectra in the fragmentation region do?

d) The slope-mass correlation and shrinking phenomena as possible universal properties of diffraction.

e) The relevance of impact-parameter models in describing structures in the differential cross-sections and the mass dependence of the helicity-flip amplitudes.

f) The dynamical connections between single and double diffraction. In particular, is factorization adequate to describe also the behaviour of differential cross-sections?

It has been known for some time that inclusive pp scattering at high energy is characterized by a large quasi-elastic peak associated with the diffractive production of high-mass states. It also appears that the mean mass of the diffractive peak and the multiplicty are correlated, the dominant contribution occuring for the two and four-prong events.

It is interesting to ask what we can learn from the frequency distribution of the different topologies and from the energy, mass and momentum transfer dependences of the process on the dynamics of diffraction.

The Pisa-Stony Brook 4π-hodoscope counter system at the ISR was placed in coincidence with the CERN-Roma small-angle magnetic spectrometer, which accepts negative particles and neutrons at essentially 0^{o} with

$0.4 < x < 0.8$. The Collaboration studied the semi-inclusive reaction

$$p + p \longrightarrow \begin{bmatrix} n \\ \pi^- \\ K^- \\ \bar{p} \end{bmatrix} + \text{n charged tracks},$$

measuring rapidity correlations and associated multiplicities as a function of the detected particle and its reduced longitudinal momentum.

The trigger, which selected interactions containing a high-momentum particle going in the direction of one of the incident protons, provides a sample of typical diffractive-like interactions.

Figure shows the multiplicity distributions associated with a π^-, K^-, $\bar{p}$, or n, the latter selected in different x intervals. A peak is observed at $n_{ch} = 2$ when a neutron is detected; the peak becomes more prominent at high x. Similarly a peak is present for $n_{ch} = 4$ when the detected particle is a π^-. Smooth distributions are obtained if the requirement of the "leading" particle is removed from the trigger. A possible explanation of the peaks is in terms of a process of the type :

$$p + p \rightarrow p + C^+$$
$$\hookrightarrow n\pi^+,\ p\pi^+\pi^-,\ K^-,\ \bar{p}$$

where C^+ indicates a cluster of particles. These "leading" clusters have the same charge and baryonic number as the incoming protons and are typical of low-multiplicity events. For neutrons with $n_{ch} = 2$ and π^- with $n_{ch} = 4$, effective cluster masses peaking around 1.5 GeV and mass-correlated slopes were observed which are suggestive of diffractive excitation, with kaons and antiprotons being produced in more central collisions with higher multiplicities.

Measurements of diffractive excitation of protons are reported by two groups at FNAL using the missing-mass approach. Precision measurements of M_X^2 and t were possible by using arrays of solid-state detectors for the slow recoil protons. The results can be divided, for ease of comparison, into two general classes; the low-mass or resonance region, typically $M_X^2 < 4\ (\mathrm{GeV})^2$, and the higher-mass interval extending up to about 50 $(\mathrm{GeV})^2$ (or roughly 0.1s).

In the resonance region the Dubna-FNAL-Rockefeller-Rochester Collaboration carried out measurements on the internal jet target, both on hydrogen and on deuterium. In the latter case, by observing the recoil deuteron, they isolate the pure isoscalar exchange component in the proton excitation. The missing-mass spectra for the reaction

$$p + p \rightarrow p + X \qquad (3)$$

for $0.01 < |t| < 0.05$ and $P_L = 175$, 260 and 400 GeV/c ($p_L = p_{lab}$) show that the cross-sections at fixed t appears rather energy-independent; a sharp peak at $M_X^2 \sim 1.3$ GeV falls very rapidly as $|t|$ increases. Breit-Wigner fits isolate a N(1400) whose differential cross-section is in good agreement with lower-energy data.

A more complex structure is seen in the reaction

$$p + d \rightarrow d + X \qquad (4)$$

owing to the excellent mass resolution, $\Delta M_X^2 = 0.07\ (\mathrm{GeV})^2$ on the average. The price to pay for using the deutron as a target lies in the complication of the deuteron form factor. The elastic pd t-distribution is damped by a factor exp $(-40\ |t|)$ which contains the deuteron form factor, the nucleon-nucleon slope, and a small Glauber contribution.

In order to obtain the proton-nucleon cross-section it is assumed that the deuteron inelastic cross-section factorizes in the same way as the elastic:

$$\frac{d^2\sigma}{dt\,dM_x^2}(p+d \rightarrow X+d) = \left[\frac{d^2\sigma}{dt\,dM_x^2}(p+p \rightarrow X+p)\right] F_d(P_L,t)$$

where $$F_d(P_L,t) = \left[\frac{\sigma_T\, pd}{\sigma_T\, pp}\right] e^{26.4t+62.3t^2}$$

This factorization hypothesis is successfully tested by comparing the pd results at P_L = 180 and 275 GeV/c with corresponding data on $p + p \rightarrow p + X$ at the same t. At both energies the agreement is very good.

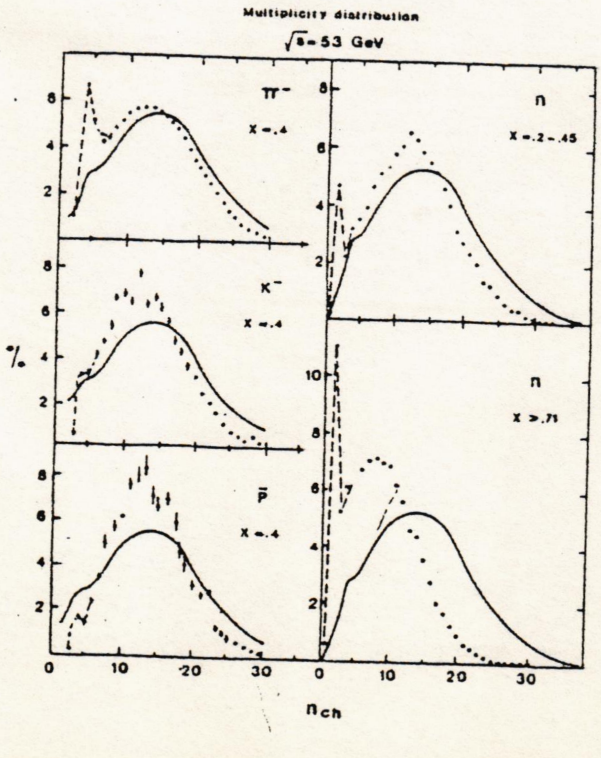
If background is taken into account, the general properties of the cross-sections change very little between 20 and 300 GeV : structures appear at

$M^2_X = 1.9$ GeV^2 and $M^2_X = 2.8$ GeV^2, probably to be identified with the N(1400), N(1688) peaks.

The t-distributions for fixed M^2_X are exponential with no sign of the turn over at least down to $|t| \sim 0.03$ (GeV/c)^2. The slope parameter, $b(M^2_X)$, seems to be a function only of M^2_X, independent of p_L.

In the resonance region $b(M_X)$ falls very rapidly from ~ 20 (GeV/c)$^{-2}$ to the average value of 7.1 ± 0.5 (GeV/c)$^{-2}$; the slope for the N (1400) bump is comparable to the value $b = 16.1 \pm 2.7$ of reaction (3).

In general, pp and pd data display very similar features both in the mass spectra and in the slope-mass dependence and compare very well with results at laboratory momenta below 30 GeV/c. By integrating the differential cross-sections we get estimates of the total production cross-section for the N(1400) bump. The comparison with lower energy data is consistent with a small energy dependence.



We also studied in detail

the tripple Regge behaviour for inclusive reactions. In the fragmentation region $1 \longrightarrow 3$, with a fixed M^2 and $s \to \infty$ we would expect Regge behaviour as

$$A(12 \to 3X) \xrightarrow[s\to\infty]{} \sum_i \gamma^i_{13}(t)\, \gamma^i_{2M}(t)\, \xi_i(t)\, P_{\alpha_i(t)}(\cos\theta_t) \qquad (1)$$

where
$$\xi_i(t) = \frac{e^{-i\pi\alpha_i(t)} + \tau_i}{\sin \pi\alpha_i(t)} \qquad (2)$$

is the signature factor. If we insert (1) into the optical theorem, viz.

$$f_1(p_3, s) = \frac{1}{2q_s\sqrt{s}}\, \mathrm{Disc}_X \{A(12\bar{3})\} \qquad (2')$$

we get

$$f_1(p_3, s) = \frac{1}{2q_s\sqrt{s}}\, \mathrm{Disc}_{M^2}\{A(12\bar{3} \to 12\bar{3})\} \to \frac{1}{s}\sum_{i,j} \gamma^i_{13}(t)\, \gamma^{j*}_{13}(t)$$
$$\times\, \xi_i(t)\, \xi^*_j(t)\, (\cos\theta_t)^{\alpha_i(t)+\alpha_j(t)}$$
$$\times\, \mathrm{Disc}_{M^2}\{A(i2 \to j2; t, M^2, t_{22'}=0)\} \qquad (3)$$

where $A(i2 \to j2)$ is the Reggeon-particle scattering amplitude.

Now if $s \gg M^2 \gg t \gg m^2_{1,2,3}$ then

$$\cot\theta_t \to \frac{s - M^2/2}{q_{t13}\, q_{t2M}} \xrightarrow[s \gg M_2]{} \frac{s}{2q_{t13}\, q_{t2M}} \xrightarrow[M^2 \gg t]{} \frac{s}{M^2} \qquad (4)$$

and for $M^2 \to \infty$ we can put

$$\mathrm{Disc}_{M^2}\{A(i2 \to j2)\} = \sum_K \gamma^K_{22}(0)\, \gamma^{ij,K}(t,0)\, \left(\frac{M^2}{s_0}\right)^{\alpha_K(0)} \qquad (5)$$

giving

$$f_1(p_3,s) = 16\pi^2 s \frac{d^2\sigma}{dt\,dM^2} = \frac{1}{s}\sum_{i,j,k} \gamma^i_{13}(t)\,\gamma^{j*}_{13}(t)$$

$$\times \xi_i(t)\,\xi^*_j(t)\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \gamma^k_{22}(0)\,\gamma^{ij,k}(t,0)\left(\frac{M^2}{s_0}\right)^{\alpha_k(0)}$$

$$= \frac{1}{s}\sum_{i,j,k} G^{ij,k}_{13,2}(t)\left(\frac{s}{s_0}\right)^{\alpha_i(t)+\alpha_j(t)} \left(\frac{M^2}{s_0}\right)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)} \qquad (6)$$

The Reggeons i, j have mass $t \equiv (p_1 - p_3)^2$ but k has mass $t_{22'} = 0$ since the optical theorem is for forward scattering. All the couplings and signature factors have been incorporated into $G^{ij,k}_{13,2}(t)$.

This expression is valid in the so-called 'triple-Regge' limit when M^2 and $s/M^2 \longrightarrow \infty$. However, this is really a misnomer because, s/M^2 gives the angle between the planes containing $1\bar{3}$ and $\bar{2}3$, and letting this angle tend to infinity is really a helicity limit. However, the leading helicity pole occurs at $\lambda = \alpha$ so the fact that we are taking a mixed Regge-helicity pole limit in (6) does not make any difference to the formula to leading order in M^2.

We know that $s/M^2 \longrightarrow \infty$ implies that $x_3 \longrightarrow 1$, $y_3 \longrightarrow y_{3\,max}$, so this triple-Regge region is only a small part of the x_3 or y_3 plot near the kinematical limit. Clearly (6) can only be applied for large s since if we suppose that we need $M^2/s > 10$, and $s/M^2 > 10$ for the Regge expansion to be valid, with $s_0 = 1\ \mathrm{GeV}^2$ this means $s > 100\ \mathrm{GeV}^2$.

Equation (6) can be rewritten

$$f_1(p_3,s) = \frac{1}{s}\sum_{i,j,k} G^{ij,k}_{13,2}(t)\,(1-x)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)} \left(\frac{s}{s_0}\right)^{\alpha_k(0)} \qquad (7)$$

and if M^2 is sufficiently large that only P is needed in the sum over K, and if the leading i and j trajectory with the quantum numbers of $1\bar{3}$ is denoted by i, then

$$f_1(p_3, s) \to \frac{1}{s} |\gamma^i_{13}(t)|^2 |\xi_i(t)|^2 \gamma^P_{22}(0) \gamma^{ii,P}(t,0) \left(\frac{s}{M^2}\right)^{2\alpha_i(t)-1} \left(\frac{s}{s_0}\right)$$

$$\sim \left(\frac{s}{M^2}\right)^{2\alpha_i(t)-1} = (1-x)^{1-2\alpha_i(t)} \qquad (8)$$

so f_1 is a function of x, or M^2/s, only, which again corresponds to Feynman scaling. And by looking at the s variation at fixed M^2, or the M^2 variation at fixed s, for different values of t, one can determine $\alpha_i(t)$ directly.

Rather comprehensive sets of fits of (6) to the high energy data have been made by Roy and Roberts and Field and Fox. In $pp \to pX$, since $1\bar{3} = p\bar{p}$ has the quantum numbers of the vacuum the leading term will be the triple-Pomeron term

$$f_1^{PPP}(p_3, s) = \frac{1}{s} G^{PP,P}_{PP,P}(t) \left(\frac{s}{s_0}\right)^{2\alpha_P(t)} \left(\frac{M^2}{s_0}\right)^{\alpha_P(0)-2\alpha_P(t)} \qquad (9)$$

which with $\alpha_P(t) \approx 1+\alpha'_P(t)$ gives

$$f_1^{PP,P} \approx \frac{1}{s_0} G^{PP,P}_{PP,P}(t) \left(\frac{s}{M^2}\right)^{1+2\alpha'_P t} \qquad (10)$$

or, also from (6),

$$\frac{d^2\sigma}{dt\,dM^2} \approx \frac{G^{PP,P}_{PP,P}(t)}{16\pi^2 s_0} \frac{s^{2\alpha'_P t}}{(M^2)^{1+2\alpha'_P t}} \qquad (11)$$

The secondary terms come from replacing i,j,k by R, where

$\alpha_R(t) \approx 0.5 + \alpha'_R t$ so we can write

$$f_1 = f_1^{PP,P} + f_1^{RR,P} + f_1^{PP,R} + f_1^{RR,R} \qquad (12)$$

where for example

$$f_1^{RR,P} = \frac{1}{s} G^{RR,P}_{PP,P}(t) \left(\frac{s}{s_0}\right)^{2\alpha_R(t)} \left(\frac{M^2}{s_0}\right)^{\alpha_P(0)-2\alpha_R(t)}$$

$$\approx \frac{1}{s_0} G^{RR,P}_{PP,P}(t) \left(\frac{s}{M^2}\right)^{2\alpha'_R(t)} \qquad (13)$$

The terms in (12) all have i=j. There could also be cross terms like $f^{PR,P}$ which are usually neglected.

Clearly, by taking different types of particle for 3 one can examine a wide range of quantum numbers for $i = 1\bar{3}$: charge exchange, strangeness exchange, baryon exchange, etc. So far, only a limited amount of data is available but some fits have been made.

Though the method is only directly applicable for $s > 100$ GeV^2 we can extend it to lower values using duality arguments. Thus at low M^2 we can expect resonances (r) to be produced which will be dual to α_K ($k = R$) in the $i2 \rightarrow j2$ amplitudue. So we expect for $i = j$ in (6)

$$\left\langle \frac{d\sigma}{dt} \right\rangle^{r} \sim \left(\frac{M^2}{s_0}\right)^{\alpha_R(0)-2\alpha_i(t)} \sim (M^2)^{(\alpha_R^0 - 2\alpha_i^0)} e^{-2\alpha_i' t \log(M^2/s_0)} \qquad (14)$$

for linear trajectories. This tells us how the differential cross-section in the two-body process $1 + 2 \rightarrow 3 + X$ should vary with M_X^2 at fixed s: it should broaden in t as M^2 increases. This triple-Regge behaviour constrains quasi-~~two~~-body scattering as well.

In the triple-Regge fits to $pp \rightarrow pX$ it is always found that, for small t, $G^{PP,P}(t) \ll G^{RR,P}(t)$ but both are non-zero for $t = 0$. The precise value depends on the assumptions made about the secondary terms, but there is now fairly general agreement about this result. Since $\gamma_{PP}^{P}(t)$ is known from fits to the pp differential cross-section this gives $\gamma^{PP,P}(t,0)$ directly. Then if at a given fixed value of t we take out the factors $\gamma_{PP}^{P}(t)$, $\xi_P(t)$ and $(s/M^2)^{\alpha_P(t)}$, corresponding to the couplings and propagators of the Reggeons i,j the remainder gives (from (5)) and the optical theorem:

$$\sigma^{tot}_{PP}(M^2,t) \rightarrow \sum_K \gamma^K_{22}(0)\, \gamma^{PP,K}(t,0) \left(\frac{M^2}{s_0}\right)^{\alpha_K(0)-1}, \quad K = R, P, \ldots \qquad (15)$$

(where we have taken s_0 / M^2 as the flux factor) which is the total cross-section for Pomeron-proton scattering as a function of 'energy', M, and the $(\text{mass})^2$ of the Pomeron, t. This leads to the result that at large M^2 $\sigma^{tot}_{PP} \rightarrow 1$ mb for $t \rightarrow 0$. Compared with $\sigma^{tot}_{PP} \simeq$ 40 mb this shows that the triple-Pomeron coupling $\gamma^{PP,P}(0,0) \approx \frac{1}{40}\gamma^{P}_{PP}(0)$, so Pomerons couple much more weakly to themselves than they do to other particles. But the coupling is not zero.

This raises a rather difficult point about the self-consistency of P exchange. The diffractive cross-section for $1+2 \rightarrow 3+X$ (with $i = P$) is, from (6):

$$\frac{d^2\sigma}{dt\,dM^2} = \frac{G^{PP,P}_{13,2}(t)}{16\pi^2 s_0^2} \left(\frac{s}{s_0}\right)^{2\alpha_P(t)-2} \left(\frac{M^2}{s_0}\right)^{\alpha_P(0)-2\alpha_P(t)} \qquad (16)$$

So if we put $\alpha_P(t) = \alpha^0_P + \alpha'_P t$ the total diffractive contribution is given by

$$\sigma^D_{12}(s) = \frac{s^{2\alpha^0_P-2}}{16\pi^2 (s_0)^{\alpha^0_P}} \int_\epsilon^s \frac{dM^2}{(M^2)^{\alpha^0_P}} \int_{-\infty}^0 dt\, G^{PP,P}_{13,2}(t)\, e^{2\alpha'_P t \log\left(\frac{s}{M^2}\right)} \qquad (17)$$

The boundary $M^2 = s$ is where $x = 1$, and ϵ marks the lower limit below which the triple-Regge approximation breaks down. Then putting say $G^{PP,P}_{13,2}(t) = G\, e^{at}$ for simplicity

$$\sigma^D_{12}(s) = \frac{G\, s^{2\alpha^0_P - 2}}{16(\pi^2)(s_0)^{\alpha^0_P}} \int_\epsilon^s \frac{}{(M^2)^{\alpha^0_P}(a + 2\alpha'_P \log(s/M^2))} \tag{18}$$

$$\sim s^{2\alpha^0_P - 2}$$

if $\alpha^0_P < 1$. But if $\alpha^0_P = 1$, using $\int \frac{dx}{x \log x} = \log(\log x)$ (19)

we find $\sigma^D_{12}(s) \propto \frac{1}{2\alpha'_P} \log\left(1 + \frac{2\alpha'_P}{a} \log s\right) \sim \log(\log s)$ (20)

Though this behaviour is compatible with the Froissart bound there is evidently an inconsistency because $\alpha^0_P = 1$ gives

$$\sigma^{tot}_{12}(s) \longrightarrow \text{constant} - O((\log s)^{-1})$$

and clearly we must have $\sigma^D_{12}(s) < \sigma^{tot}_{12}(s)$ as $s \longrightarrow \infty$.
Indeed no ordinary Regge singularity can give $\sigma^{tot} \sim \log$ (logs).
On the other hand if $G^{PP,P}_{13,2}(t)$ vanished at $t = 0$, for example

$$G^{PP,P}_{13,2}(t) = (-t)\, G\, e^{at}$$

say, then (17) would give

$$\sigma^D \propto \int_\epsilon^s \frac{dM^2}{(M^2)^{\alpha^0_P}\left(a + 2\alpha'_P \log\left(\frac{s}{M^2}\right)\right)^2} \propto \frac{1}{2\alpha'_P a} - \frac{1}{2\alpha'_P(a + 2\alpha'_P \log s)}$$

$$\rightarrow \text{constant} - O((\log s)^{-1}) \tag{21}$$

which would be compatible with P dominance. This problem, first noted in the context of the multi-peripheral model by Finkelstein and Kajantie has been re-examined by many authors. We intend to study this problem further.